

# Systematic Software Analysis Using SAT

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# Overview

SAT solvers have many uses, e.g., model finding, model enumeration, and model counting

This lecture focuses on **model enumeration**

It has many applications in software (and hardware) engineering

- **Testing:** create high quality test suites
- **Analysis:** illustrate different counterexamples
- **Synthesis:** create alternative implementations
- **Repair:** create alternative fixes



# An application of enumeration

## Systematic testing of code using specs [ASE'01]

- Idea: create **all** “small” inputs, and test against them
  - High quality suites with non-equivalent inputs
    - **Symmetry breaking** [SAT'03]
- Enabling technology: Alloy tool-set [Jackson-FSE'00]
  - Alloy: relational first-order logic + transitive closure
  - Alloy analyzer: SAT-based tool for automatic analysis
  - <http://alloy.mit.edu>



# Outline

Basics of software testing

- Focus: programs with structurally complex inputs

Basics of Alloy

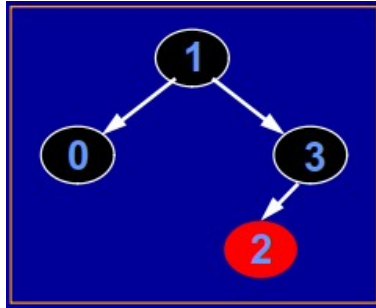
Basics of systematic testing

- Create non-equivalent tests using symmetry breaking

Conclusions

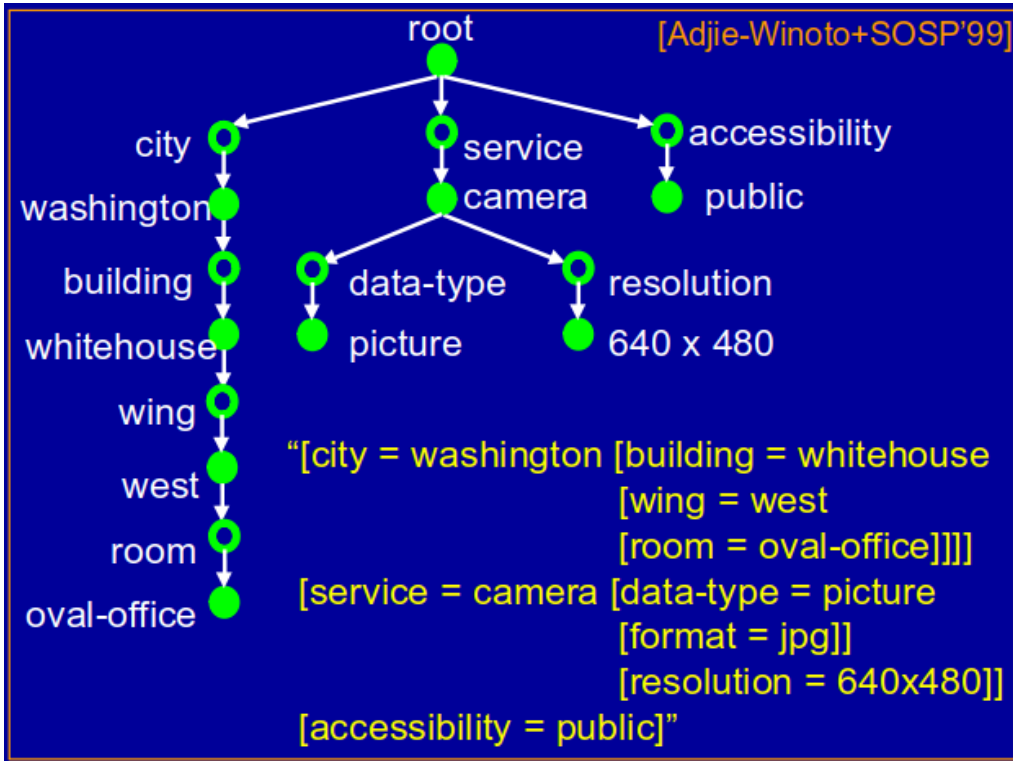


# Structurally complex data



```

.ClassName4{
    -webkit-transform: rotateY( 180deg );}
.ClassName12{
    -webkit-perspective: 800;
    -webkit-backface-visibility: hidden;}
  
```



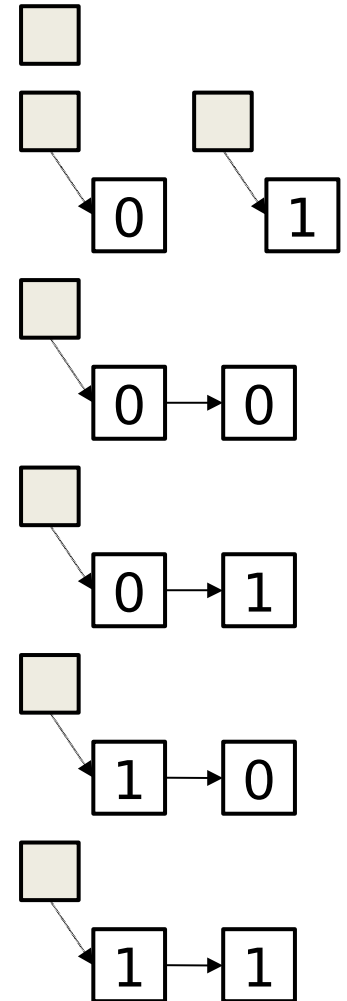
```

<html>
<head>
<link rel="stylesheet"
type="text/css" href="file.css">
</link></head>
<body>
<div class="ClassName4">
<h1>This is some text
<div class="ClassName12">
<h1>This is some text</h1>
</div></h1></div></body></html>
  
```



# Acyclic singly-linked list

```
class SLList {  
    // class invariant: acyclic and size-okay  
    Node header;  
    int size;  
  
    static class Node {  
        int elem;  
        Node next; }  
  
    void add(int x) {  
        // pre-cond: class invariant (this)  
        // post-cond: class invariant (this)  
        //           and x is added at the head  
  
        Node n = new Node();  
        n.elem = x;  
        n.next = header;  
        header = n;  
        size++; }  
  
    void remove(int x) { /*... */ }
```



# How to create an input list?

Write a test (by hand)

Two basic ways: at *abstract* level or at *concrete* level

```
@Test public void abst() {  
    // create receiver object state  
    SLList l = new SLList();  
    l.add(0);  
  
    // execute method to test  
    l.remove(1);  
  
    // (partially) check output  
    assertEquals(0, l.header.elem);  
}
```

```
@Test public void conc() {  
    // create receiver object state  
    SLList l = new SLList();  
    Node n0 = new Node();  
    l.header = n0; l.size = 1;  
    n0.elem = 0; n0.next = null;  
  
    // execute method to test  
    l.remove(1);  
  
    // (partially) check output  
    assertEquals(0, l.header.elem);  
}
```



# How to create many lists?

Can write a test generator (by hand)

Can automate using **non-deterministic choice**, e.g., with the Java PathFinder [<https://github.com/javapathfinder>]

```
static List abstractGen() {  
    List l = new List();  
    int length = Verify.getInt(0, 2);  
    for (int i = 0; i < length; i++) {  
        boolean method = Verify.getBoolean();  
        int arg = Verify.getInt(0, 1);  
        if (method) {  
            l.add(arg);  
        } else {  
            l.remove(arg);  
        }  
    }  
    return l; }  
}
```





# Abstract-level generation

Advantage: simple to automate

Disadvantage:

- Hard to test partial implementations
  - To test *remove*, must implement *add* first
- Hard to avoid equivalent tests
- E.g., naive exploration creates 21 method sequences:  
 $\epsilon$ , “add(0)”, “add(1)”, “remove(0)”, “remove(1)”,  
“add(0); add(0)”, “add(0); add(1)”, “add(0); remove(0)”,  
“add(0); remove(1)”, ...



# How to create many lists - at the concrete level?

Again, can write a test generator (by hand), or automate using non-deterministic choice

Advantage: efficient, high quality test generation

Disadvantage:

- Different structures require different generators
- Writing the generators can be hard
  - No textbook methods
  - Cannot simply sample at random:  $\#valid/\#all \rightarrow 0$
- Generators need to account for symmetry breaking

**Idea: use logical constraints and model enumeration!**

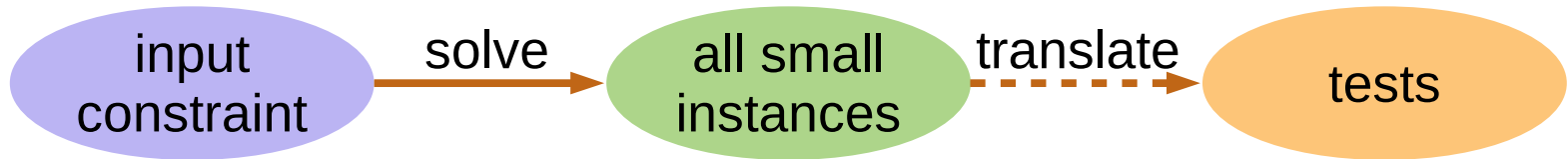


# Constraint-based generation

Observe: each input must be a **valid** structure

- **Acyclic**, singly-linked list

Approach: characterize validity properties as logical constraints, and solve them [ASE'01]



Two key questions:

- How to write the constraints?
- How to solve the constraints to find one, many, or all solutions?



# How to write constraints?

Use a declarative language, e.g., Alloy

```
pred Acyclic(l: List) {  
    all n: l.header.*link | n !in n.^link  
}
```

Use an **imperative** language, e.g., Java

```
boolean repOk() {  
    if (header == null) return size == 0;  
    Set<Node> visited = new HashSet<Node>();  
    Node current = header;  
    while (current != null) {  
        if (!visited.add(current)) return false;  
        current = current.next;  
    }  
    return size == visited.size();  
}
```



# How to solve constraints?

For Alloy, its analyzer provides fully automatic solving using off-the-shelf SAT technology

- Kodkod back-end [TorlakJackson-TACAS'07]
- Supports several SAT solvers

For Java, there are four basic approaches:

- Translate to SAT, a la bounded model checking [Biere+TACAS'99, JacksonVaziri-ISSTA'00]
- Use symbolic execution [King-CACM'76, TACAS'03]
  - For each path that returns true, create input(s)
- Filter (naively) all candidates using *repOk*
- Use a dedicated solver for Java, e.g., Korat [ISSTA'02]



# Non-det. choice and filtering

```
static void concreteGen() {  
    // allocate objects  
    SLList l = new SLList();  
    Node n1 = new Node();  
    Node n2 = new Node();  
  
    // build domain(s)  
    Node[] nodes = new Node[]{ null, n1, n2 };  
  
    // initialize fields  
    l.header = nodes[Verify.getInt(0, nodes.length - 1)];  
    l.size = Verify.getInt(0, 2);  
    n1.elem = Verify.getInt(0, 1);  
    n1.next = nodes[Verify.getInt(0, nodes.length - 1)];  
    n2.elem = Verify.getInt(0, 1);  
    n2.next = nodes[Verify.getInt(0, nodes.length - 1)];  
  
    // check validity  
    if (l.repOk()) {  
        // output list  
    }  
}
```



# Solving imperative constraints

*repOk* is a logical constraint written in an imperative language, hence termed *imperative constraint*

Solving *repOk* using naive filtering is infeasible

- Checks every candidate in the state space (e.g., 324)
- Creates too many solutions that are redundant
  - E.g., 68 valid lists (instead of 7 that we expect)

However, *repOk* can be used to **prune** the search and make it feasible [ISSTA'02]

- Korat prunes **and** checks only **non-isomomorphic** candidates (e.g., 31)
- Creates non-equivalent solutions (e.g., 7 valid lists)



# Alloy





# Alloy demo



# An Alloy specification

**module** list

**one sig** List { // set of list atoms

header: **lone** Node } // header: List x Node

**sig** Node { // set of node atoms

link: **lone** Node } // link: Node x Node

**pred** RepOk(l: List) { **all** n: l.header.\*link | n **!in** n.^link }



# Alloy: simulation

```
module list
```

```
one sig List { // set of list atoms
```

```
  header: lone Node } // header: List x Node
```

```
sig Node { // set of node atoms
```

```
  link: lone Node } // link: Node x Node
```

```
pred RepOk(l: List) { all n: l.header.*link | n !in n.^link }
```

```
run RepOk // default scope is 3
```

```
fact Reachability { List.header.*link = Node }
```



# Alloy: checking

```
sig List { header: lone Node }
```

```
sig Node { link: lone Node }
```

```
pred RepOk(l: List) { all n: l.header.*link | n !in n.^link }
```

```
pred RepOk2(l: List) {  
  no l.header or some n: l.header.*link | no n.link }
```

```
assert Equivalence {  
  all l: List | RepOk[l] <=> RepOk2[l] }
```

```
check Equivalence // for 1, 2, 3, 4, 5, 6, ...
```



# Symmetry breaking (SB)

Alloy adapts Crawford's symmetry breaking predicates to remove some, but not all, symmetries [Shlyakhter-SAT'01]

We can remove all symmetries – for a class of structures – by writing additional constraints **in Alloy** [SAT'03]

- For example:
  - Define a linear order on nodes
  - Add constraints to define a “traversal” and require the nodes to be “visited” w.r.t. the linear order



# Full symmetry breaking: lists

```
open util/ordering[Node]
```

```
module list
```

```
one sig List { header: lone Node }
```

```
sig Node {link: lone Node }
```

```
pred RepOk(l: List) { all n: l.header.*link | n !in n.^link }
```

```
fact SymmetryBreaking {  
  List.header in first[]  
  all n: List.header.*link | n.link in next[n]  
}
```



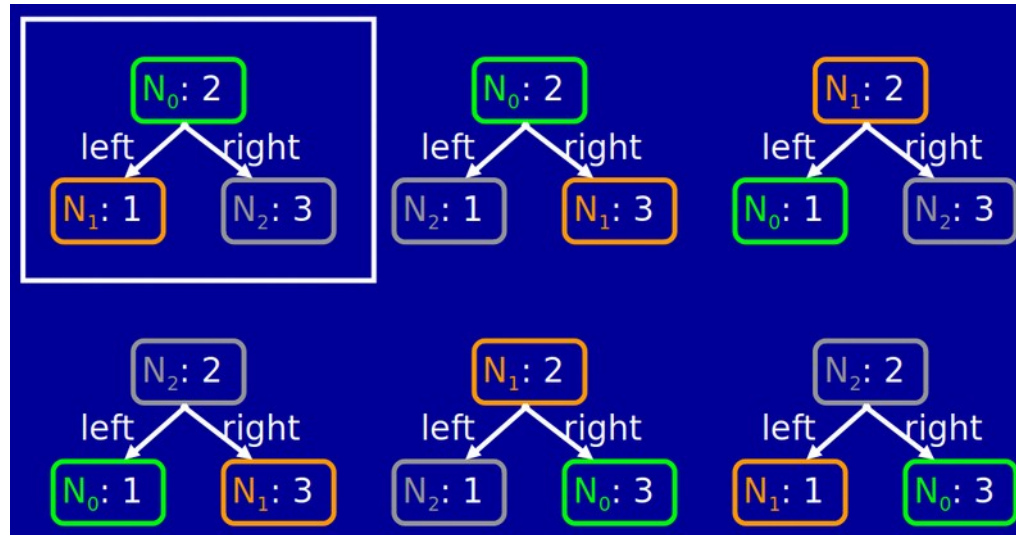
# Full symmetry breaking: binary search trees

```
fact SymmetryBreaking { // pre-order
  Tree.root in first[]
  all n: Tree.root.*(left + right) {
    some n.left implies n.left in next[n]
    no n.left implies n.right in next[n]
    some n.right and some n.left implies
      n.right in next[max[n.left.*(left + right)]]
  }
}
```



# Full symmetry breaking: illustration

For exactly 3 nodes (and integer keys  $\{1, 2, 3\}$ ), there are  $3! = 6$  trees in each isomorphism class, e.g.,



- Each permutation of node identities ( $N_0, N_1, N_2$ ) gives an isomorphic tree

With the *SymmetryBreaking* fact only 1 tree (that respects the pre-order traversal constraint) per class is generated

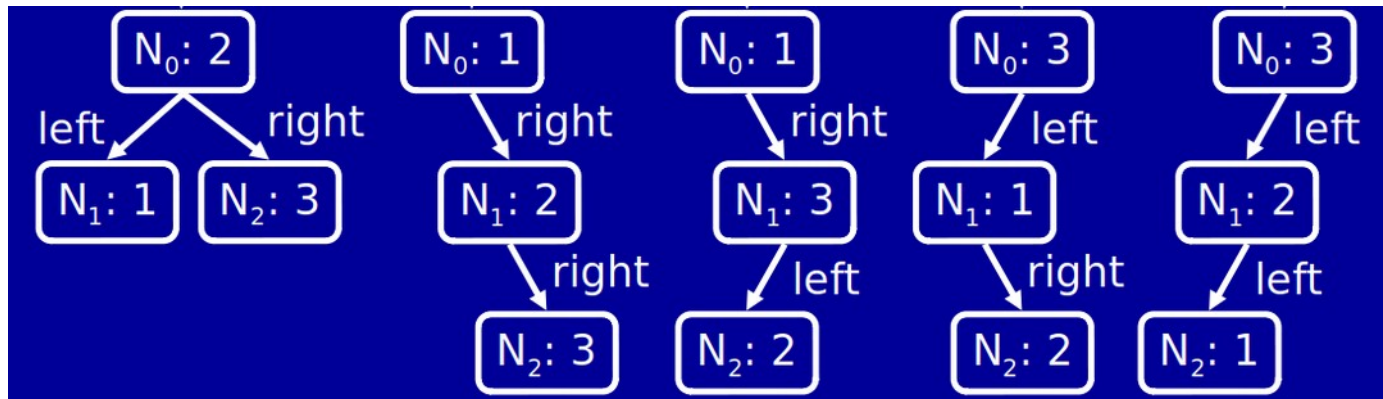


# Importance of symmetry breaking

With no symmetry breaking the number of solutions goes up by a factor that is exponential in the number of nodes

- Also, the solver can suffer a substantial slowdown

E.g., with full symmetry breaking, there are 5 trees (with 3 nodes and keys {1, 2, 3}):



- With no symmetry breaking, there are  $5 \times 3! = 30$  trees

For red-black trees with 9 nodes, solving time is  $>5x$  less for full symmetry breaking vs. Alloy's default SB [SAT'03]



# Results (historic context)

<i>benchmark</i>	<i>size</i>	<i>structures generated</i>	<i>time (sec)</i>	<i>state space</i>
<u>BinarySearchTree</u>	7	429	7	$2^{186}$
	9	4862	618	$2^{292}$
<u>HeapArray</u>	6	13139	6	$2^{72}$
	8	1005075	1172	$2^{110}$
<u>java.util.LinkedList</u>	7	877	1	$2^{191}$
	10	115975	304	$2^{362}$
java.util.TreeMap	7	35	111	$2^{263}$
	9	122	742	$2^{407}$
java.util.HashSet	7	1716	32	$2^{119}$
	9	24310	512	$2^{215}$

Using Alloy with mChaff back in the early 2000's [SAT'03]



# Related work (a few pointers)

## Solution enumeration

Alloy

Symmetry [Shlyakhter-SAT'01][KMSJ-SAT'03]

Minimality [Nelson+ICSE'13]

Field exhaustiveness [Ponzio+FSE'16]

Coverage [SPIN'14][Porncharoenwase+FM'18]

Alternative formulation [Trippel+MICRO'18]

Other  
systems

Dedicated search [BKM-ISSTA'02][KPV-TACAS'03]

Mixing solvers and dedicated generators  
[GGJKKM-ICSE'10][Kuraj+OOPSLA'15]

Solver-aided languages [Ringer+OOPSLA'17]

Sampling [Meel+AAAI-Workshop'16][Dutra+ICCAD'18]



# Conclusions

Model enumeration has many applications in software (and hardware) engineering

- E.g., in testing, analysis, synthesis, and repair

Symmetry breaking is vital for scalability!

- Without it, too many redundant solutions and much higher time cost

Designing SAT solvers for faster/better enumeration is very important!

CNF benchmarks for enumeration and symmetries:

<http://projects.csail.mit.edu/mulsaw/alloy/sat03>

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