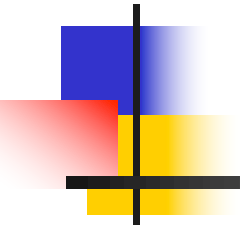
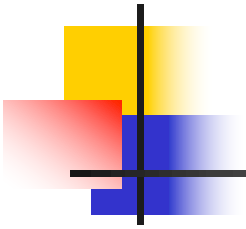


Knowledge Compilers

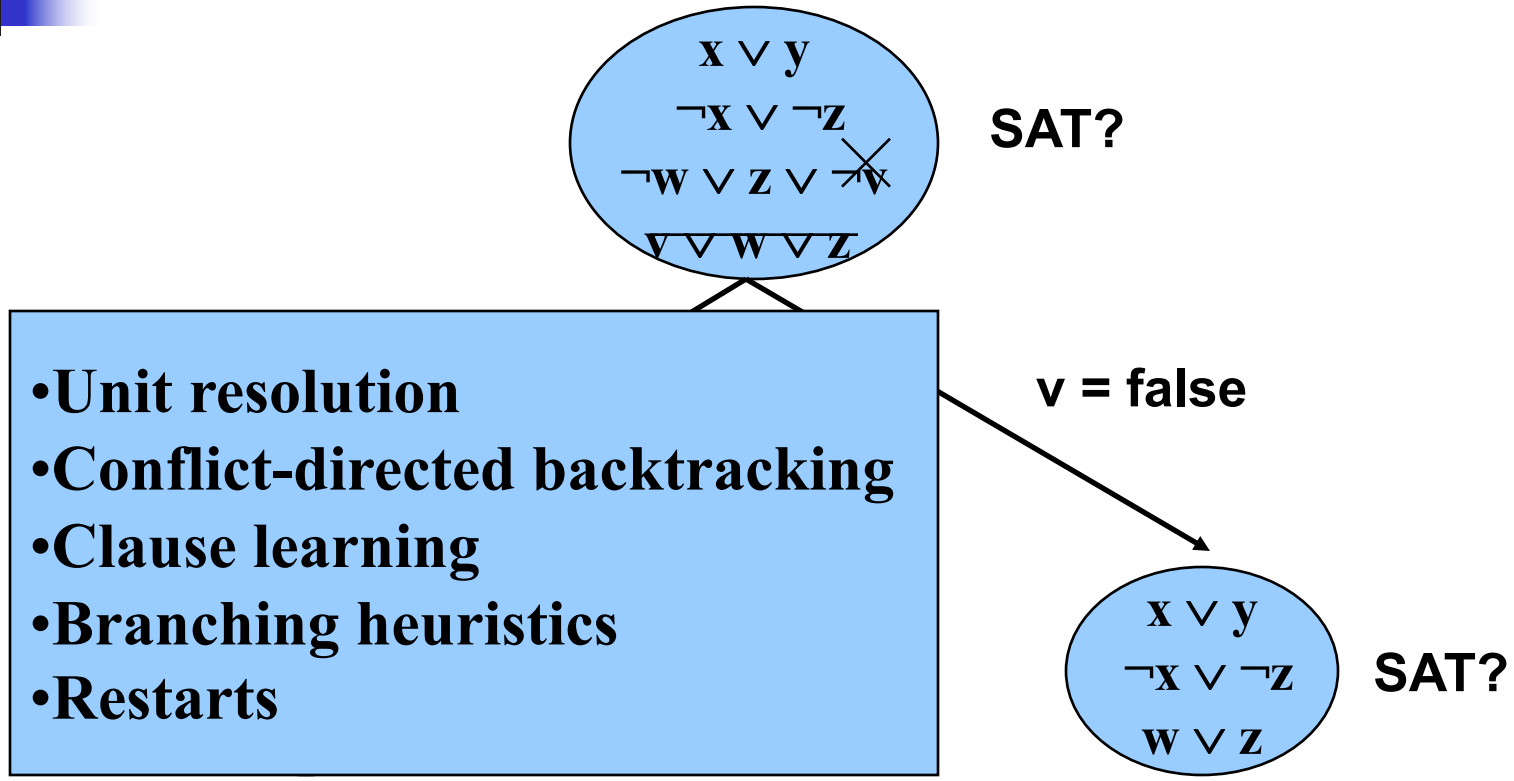
Adnan Darwiche
Computer Science Department, UCLA

Building Compilers



- To-down approaches:
  - Based on exhaustive search
- Bottom-up approaches:
  - Based on transformations

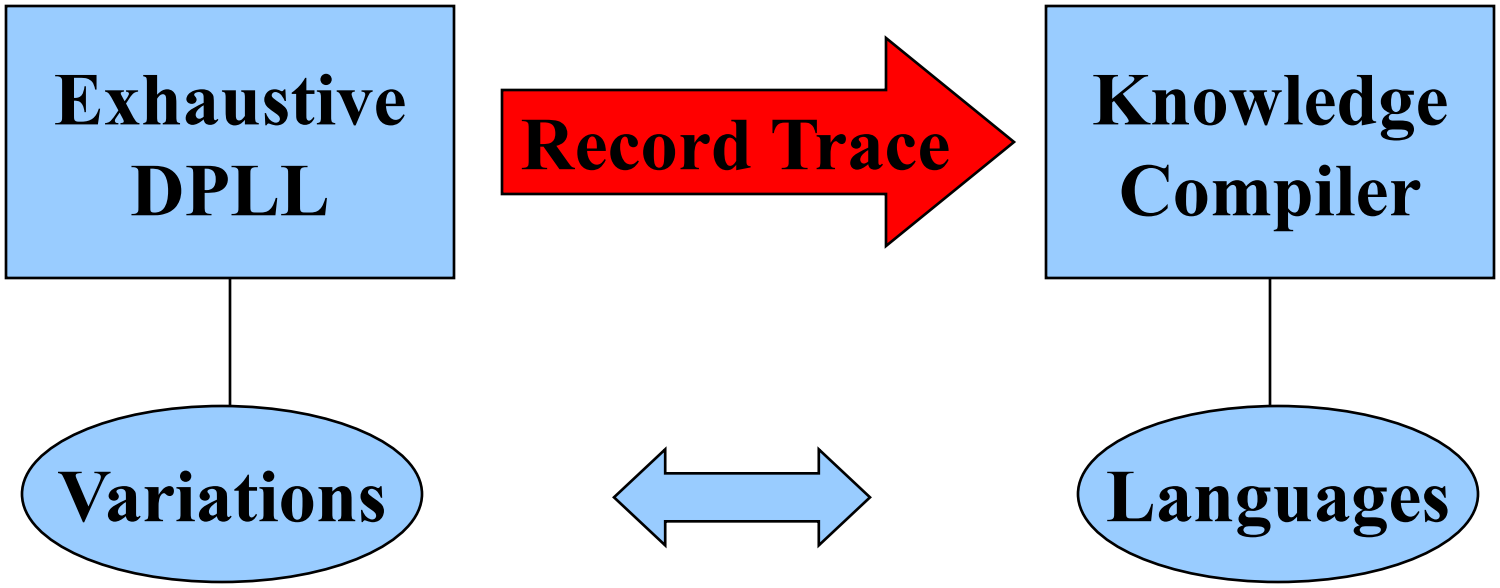
SAT by DPLL Search



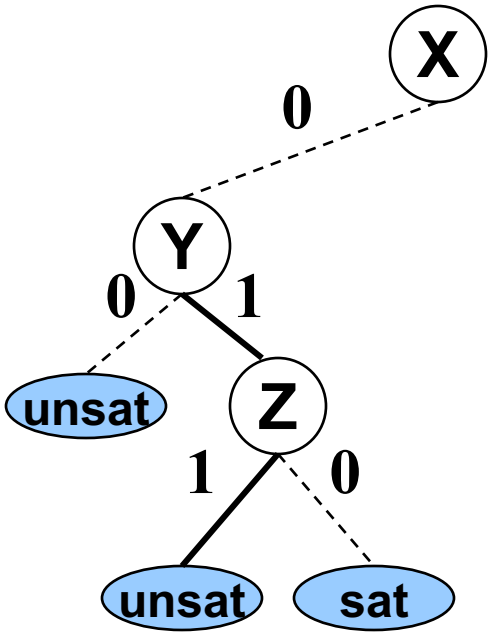
Terminating condition for recursion:

empty set (satisfied), or empty clause (contradiction)

The Language of Search



Trace of DPLL



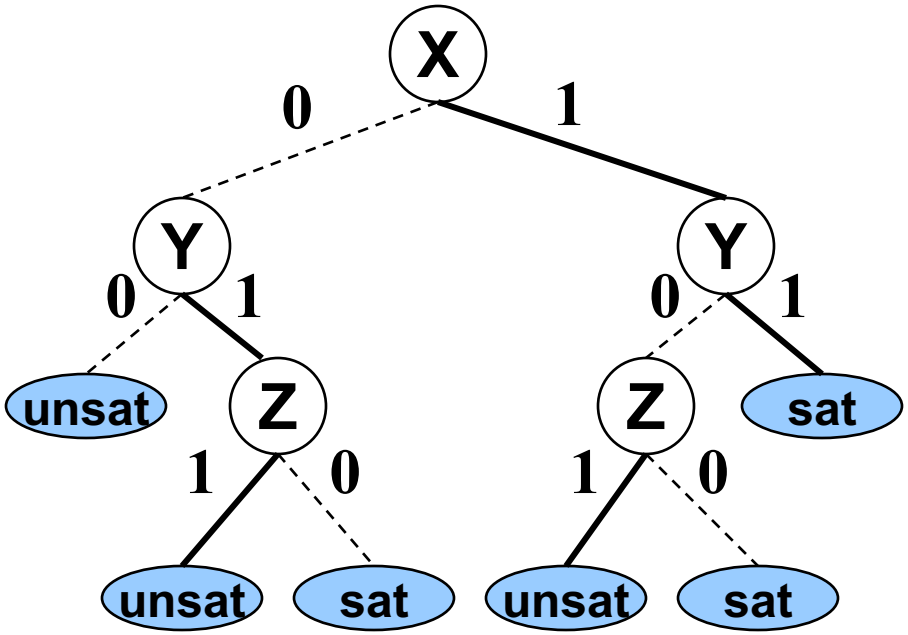
$$X \vee Y$$

$$X \vee \neg Y \vee \neg Z$$

$$\neg X \vee Y \vee \neg Z$$

Exhaustive DPLL

Run to Exhaustion

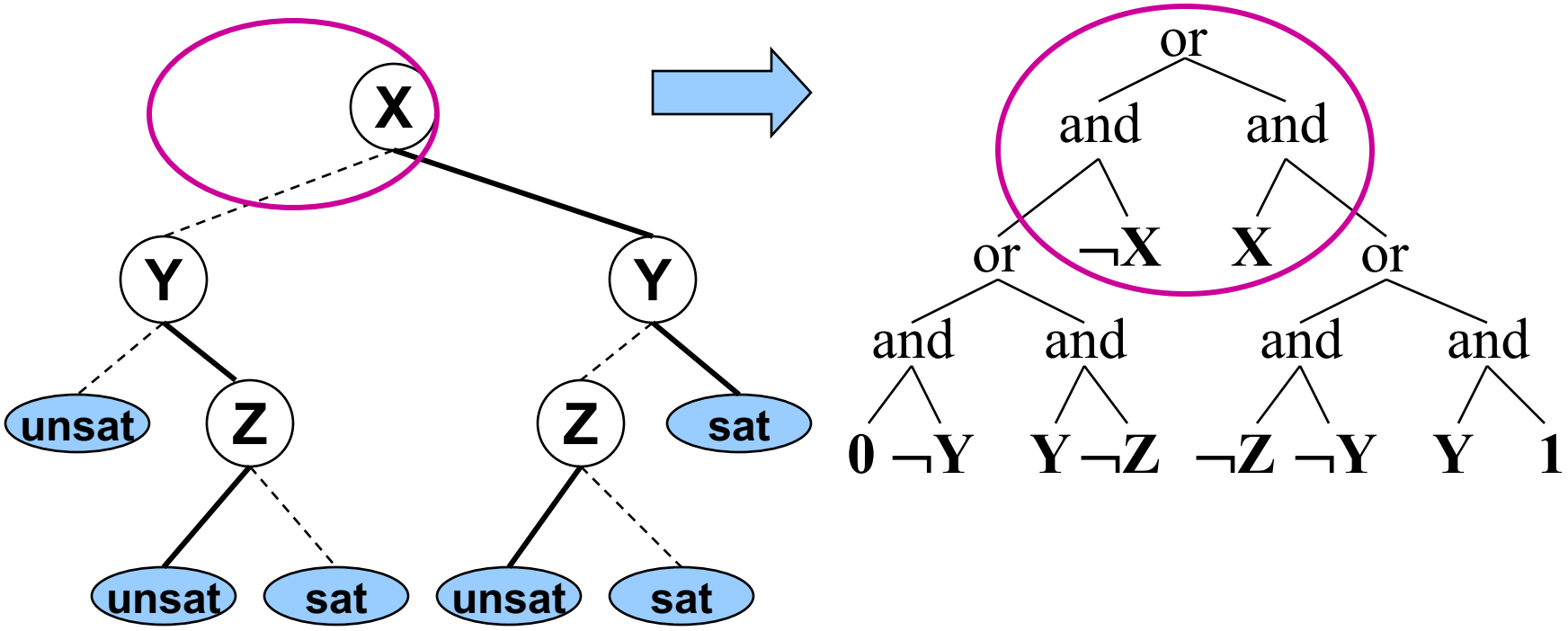


$$X \vee Y$$

$$X \vee \neg Y \vee \neg Z$$

$$\neg X \vee Y \vee \neg Z$$

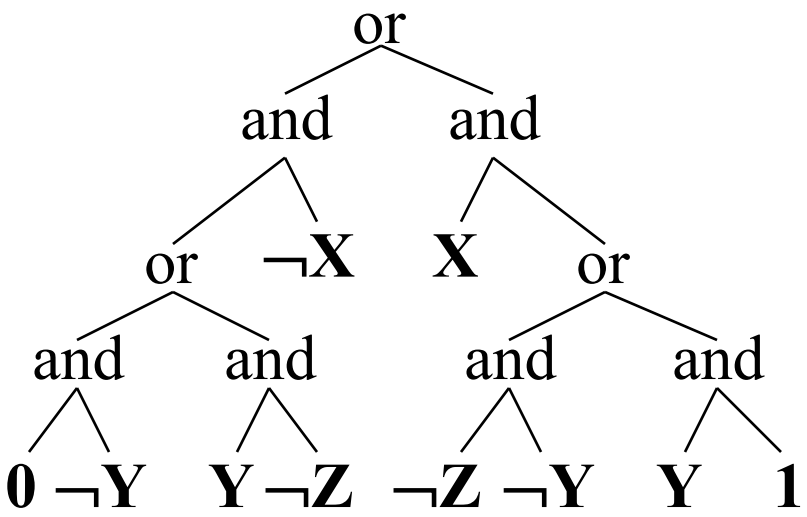
Trace of DPLL: a Formula



Trace of DPLL: a Formula

Equivalent to original CNF

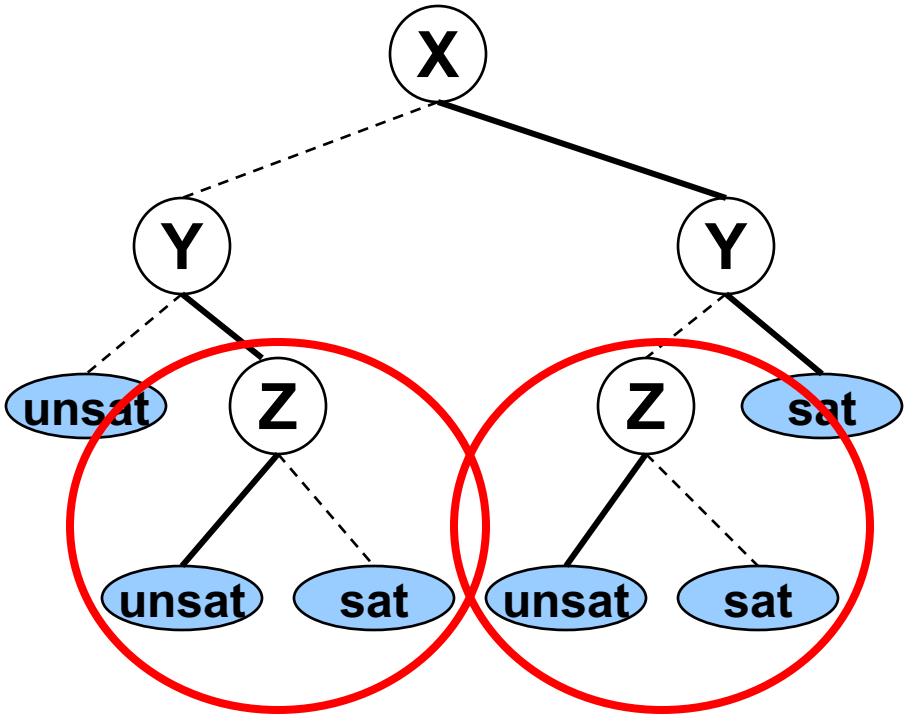
Tractable
(e.g., count models)



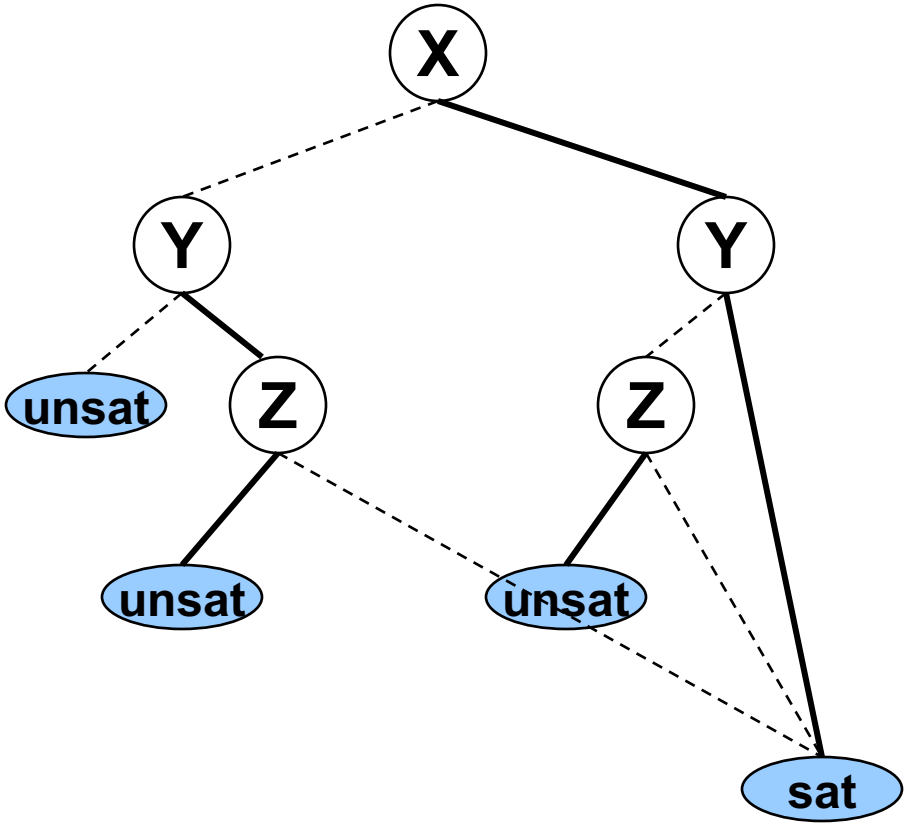
Dealing with Redundancy

Level One: Do not record redundant portions of trace

Level Two: Try not to solve equivalent subproblems

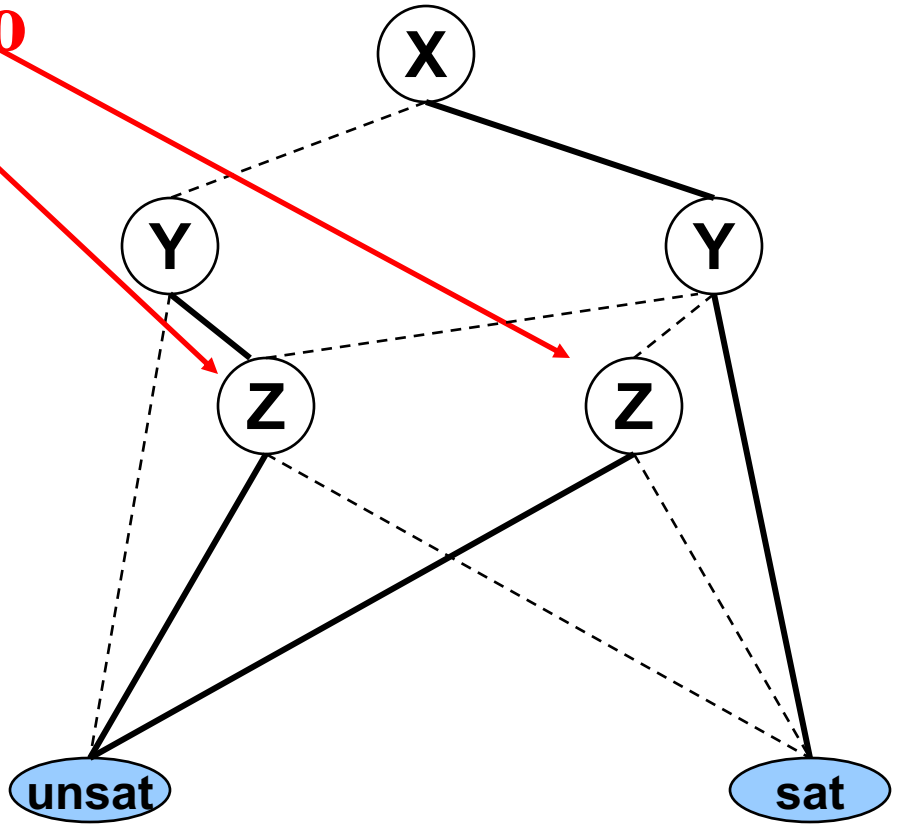


Dealing with Redundancy

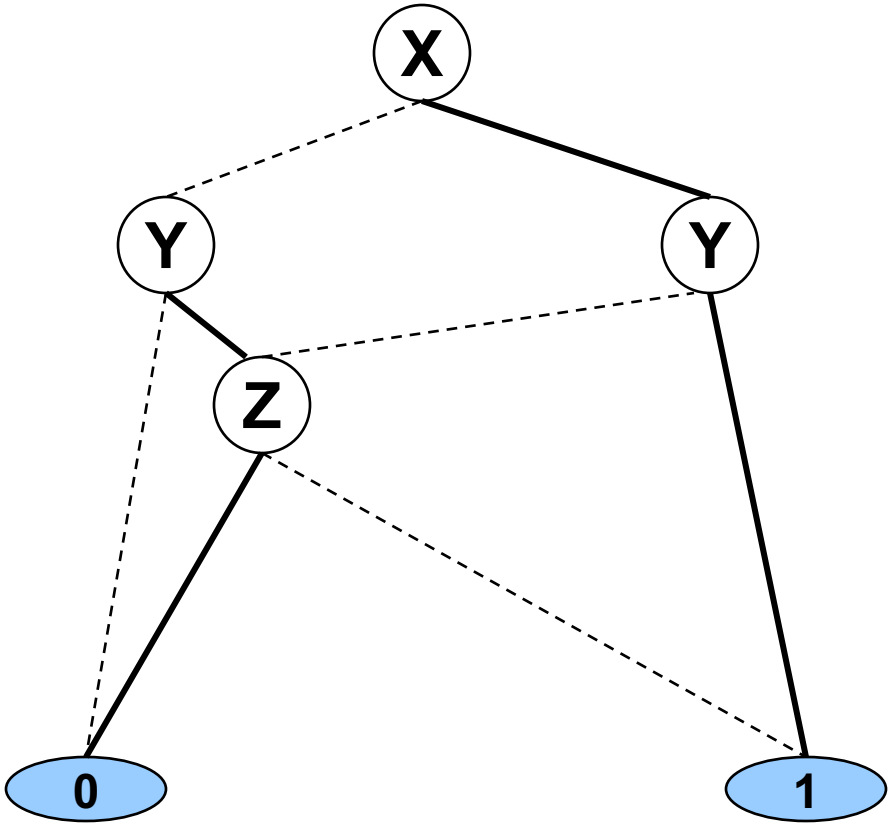


Dealing with Redundancy

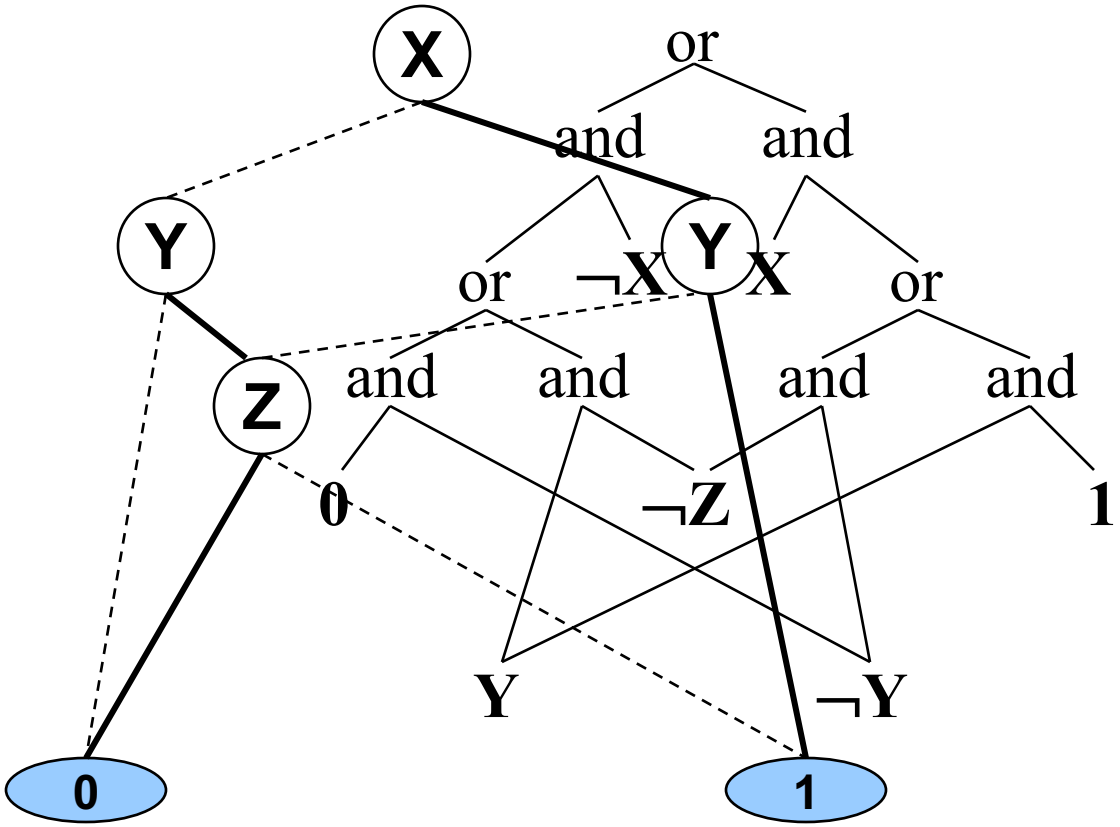
Simply create to existing node



This is an OBDD!



This is an OBDD!



NNF + decision, decomposability, ordering

A Non-traditional OBDD Compiler

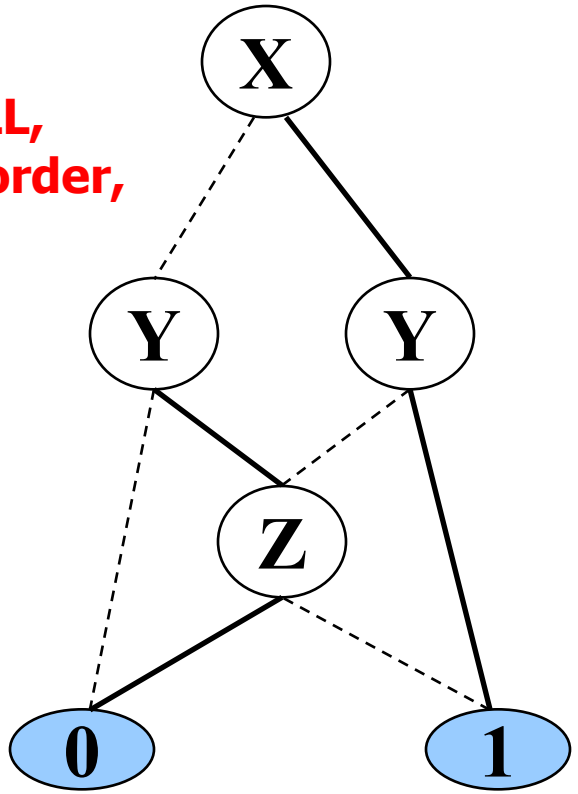
$X \vee Y$

$X \vee \neg Y \vee \neg Z$

$\neg X \vee Y \vee \neg Z$

Exhaustive DPLL,
Fixed variable order,
Unique nodes

Compile



New complexity guarantees

FBDD

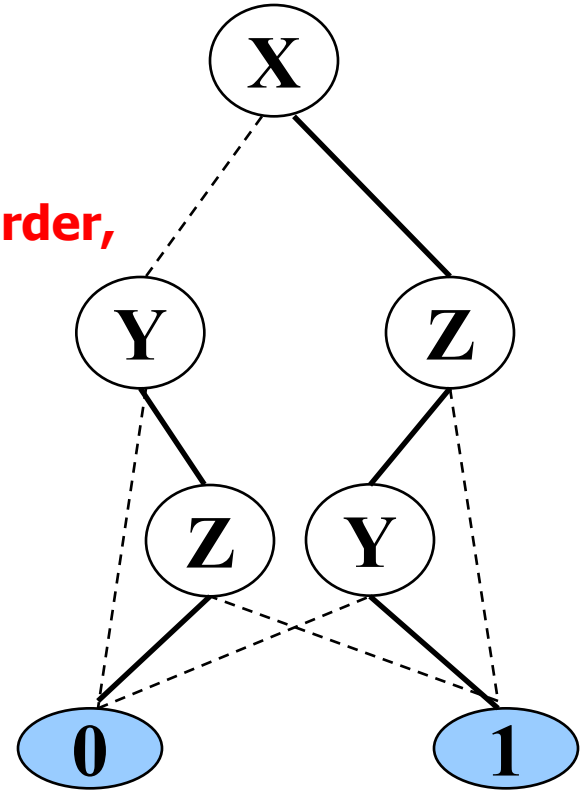
$X \vee Y$

$X \vee \neg Y \vee \neg Z$

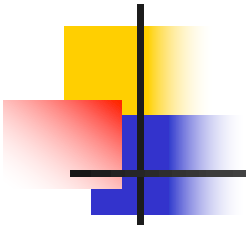
$\neg X \vee Y \vee \neg Z$

Exhaustive DPLL,
Dynamic variable order,
Unique nodes

Compile



NNF + decision, decomposability



Dealing with Redundancy

- **Level One: Unique nodes (done)**
- **Level Two: Avoid redundant compilation (searches)**

Redundant Compilation

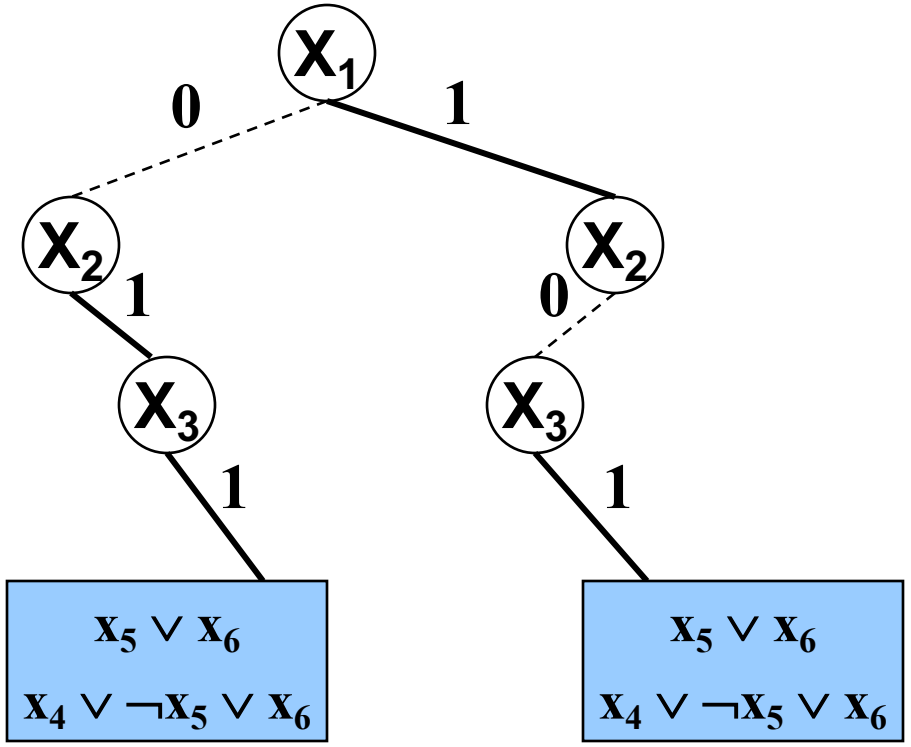
$x_5 \vee x_6$

$x_4 \vee \neg x_5 \vee x_6$

$x_1 \vee x_3 \vee x_4 \vee x_5$

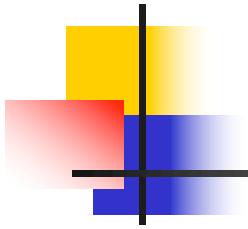
$x_2 \vee x_3$

$x_1 \vee x_2 \vee \neg x_3$



Formula Caching: complexity guarantees

Beyond BDDs…

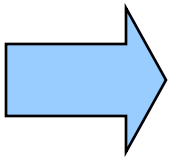


Plain DPLL → **FBDD**

Fixed Variable Ordering → **OBDD**

Decomposition (Component Analysis)

Solve disjoint subproblems independently



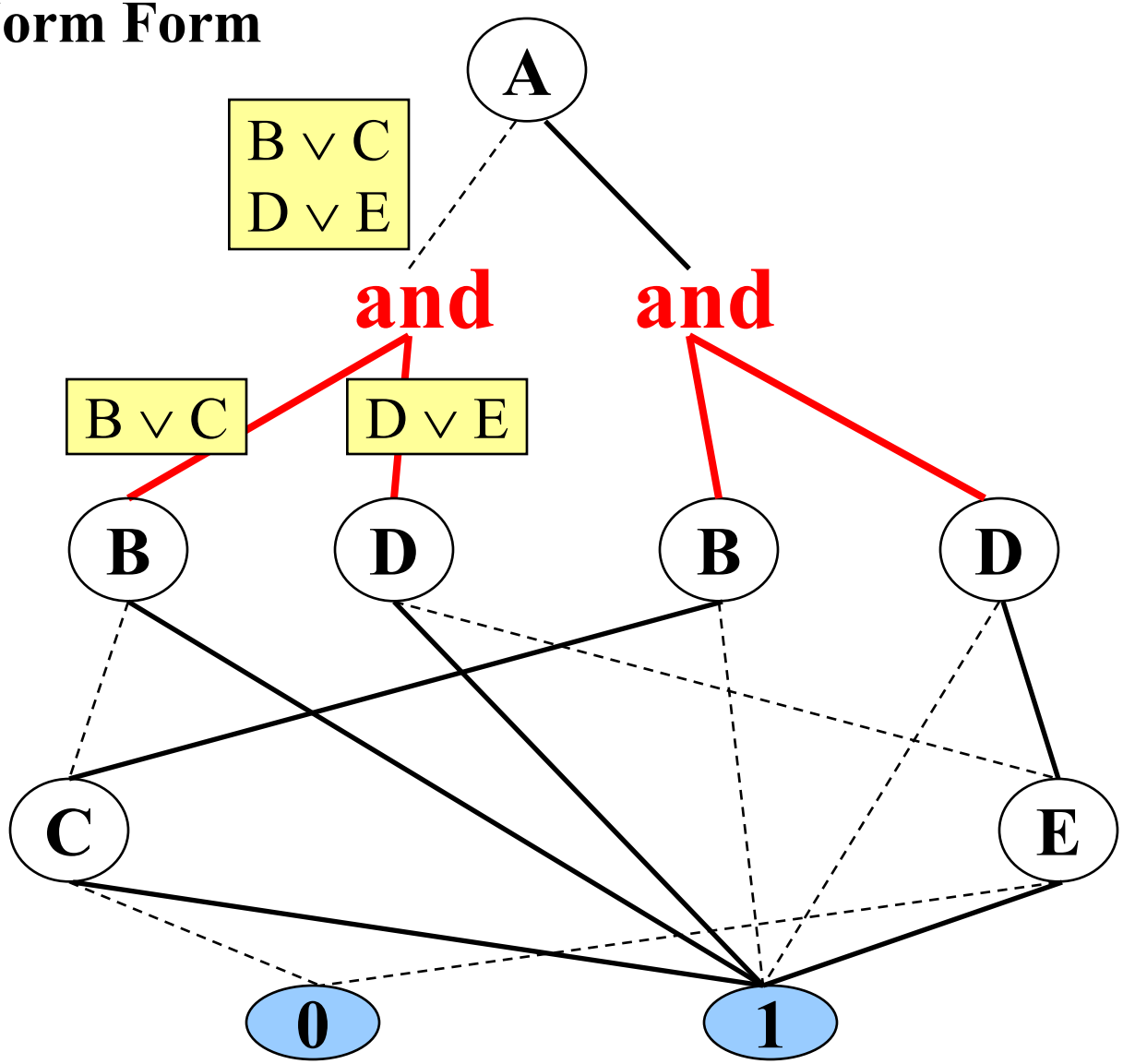
d-DNNF

Combine as AND node

Deterministic
Decomposable
Negation Norm Form

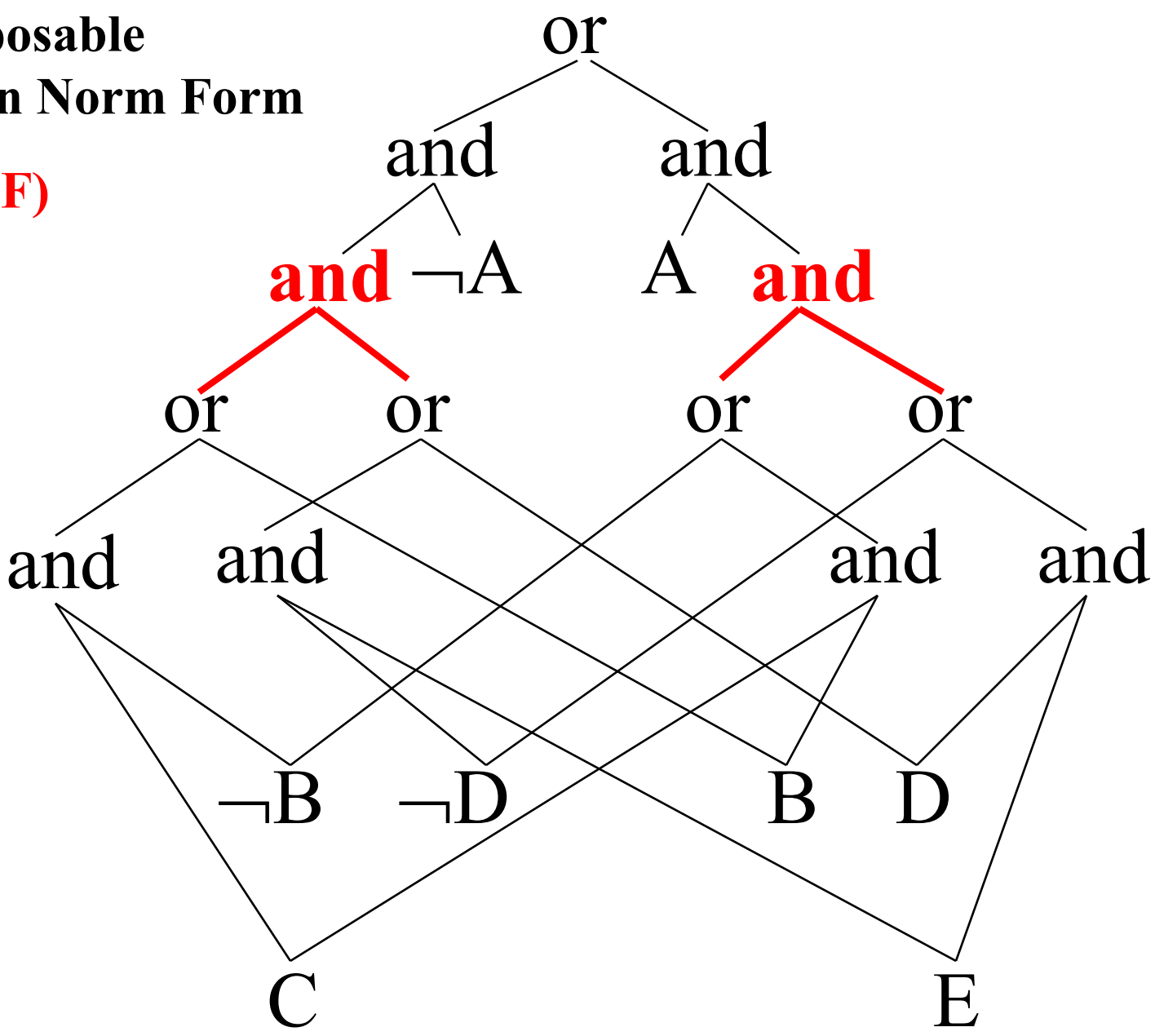
(d-DNNF)

$A \vee B \vee C \qquad \neg A \vee \neg B \vee C$
$A \vee D \vee E \qquad \neg A \vee \neg D \vee E$

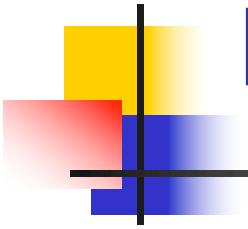


Deterministic
Decomposable
Negation Norm Form

(d-DNNF)

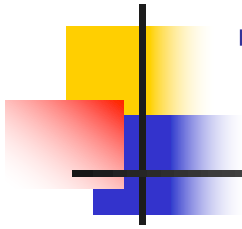


Decomposition Strategies



- Static (c2d)
  - Hypergraph partitioning pre-compilation
  - Strategy captured using a dtree (decomposition tree)
- Dynamic (Dsharp/Cachet)
  - Dynamic variable orderings
  - Lazy detection of decompositions
- Mixed (D4)
  - Hypergraph partitioning during compilation
  - Done selectively (using dtrees)

The Language of Search



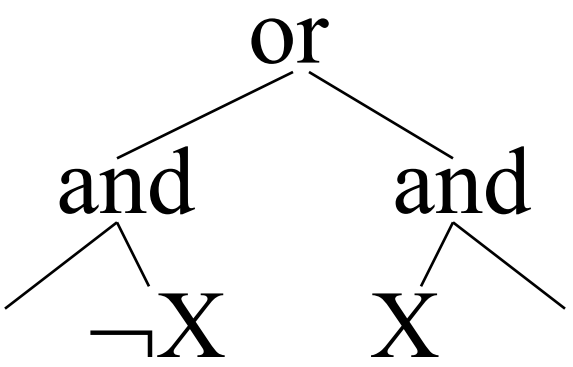
Fixed Variable Ordering → **OBDD**

Plain DPLL → **FBDD**

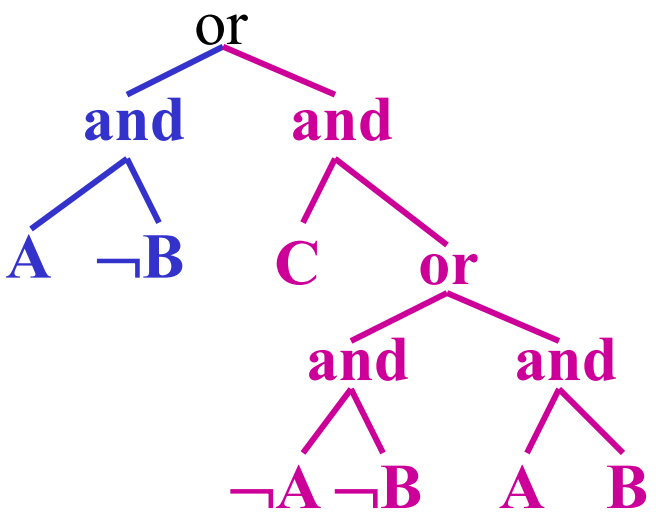
Allowing Decomposition → **d-DNNF**

Limitation of DPLL: General determinism

Decision nodes (Decision-DNNF)

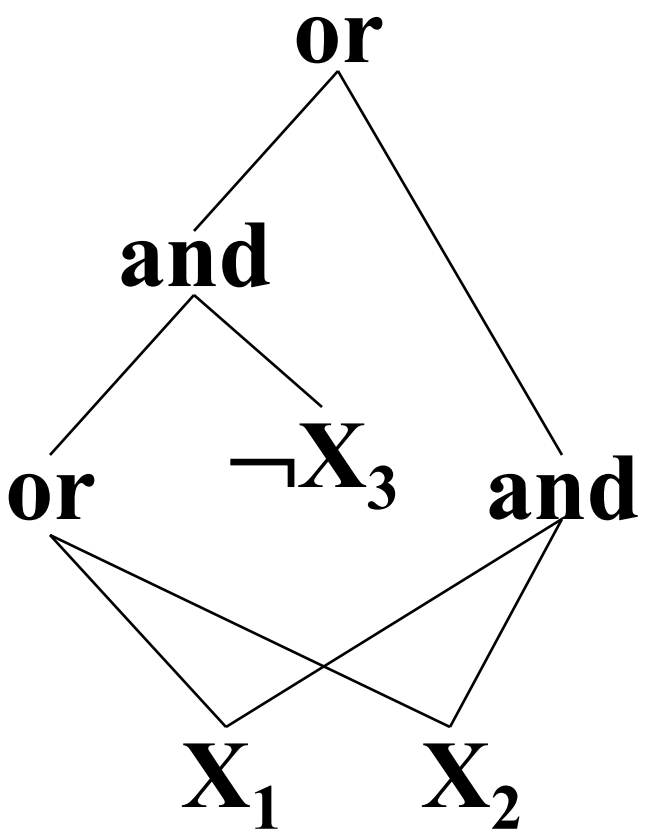


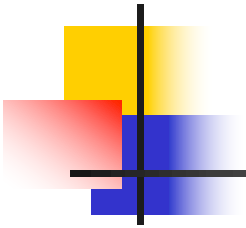
Deterministic nodes (d-DNNF)



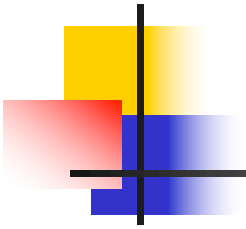
Beyond DPLL:
Decomposability (D) without determinism (d)

DNNF:
CO, CE, ME, exist quantification





Bottom-up Compilation



Bottom-up Compilation

CNF: (x + y) (y + z)

Variable order: x, y, z

Bottom-up Compilation

CNF: (x + y) (y + z)

Variable order: x, y, z

Apply:

combines two OBDDs using Boolean operators

Bottom-up Compilation

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Variable order: x, y, z

Apply:

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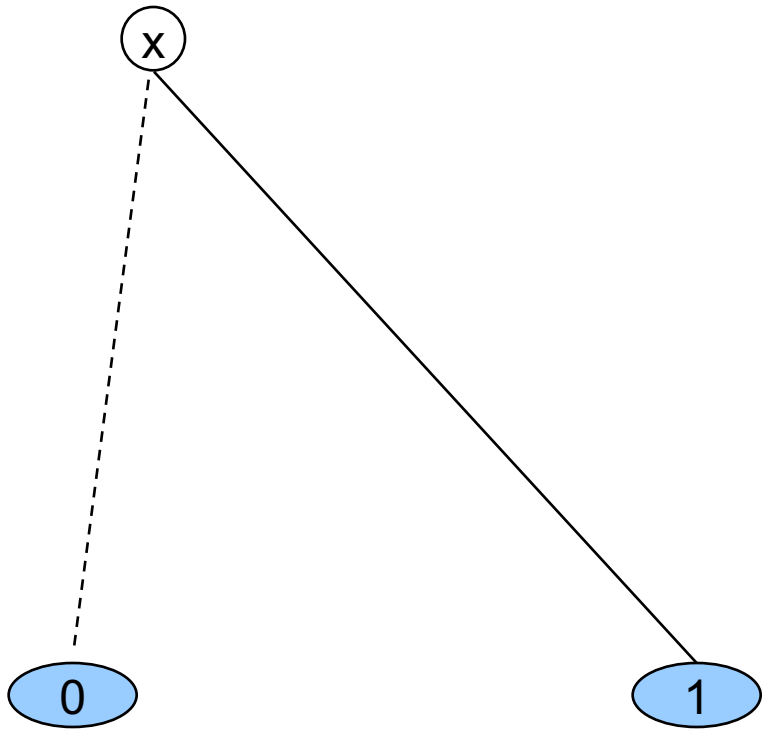


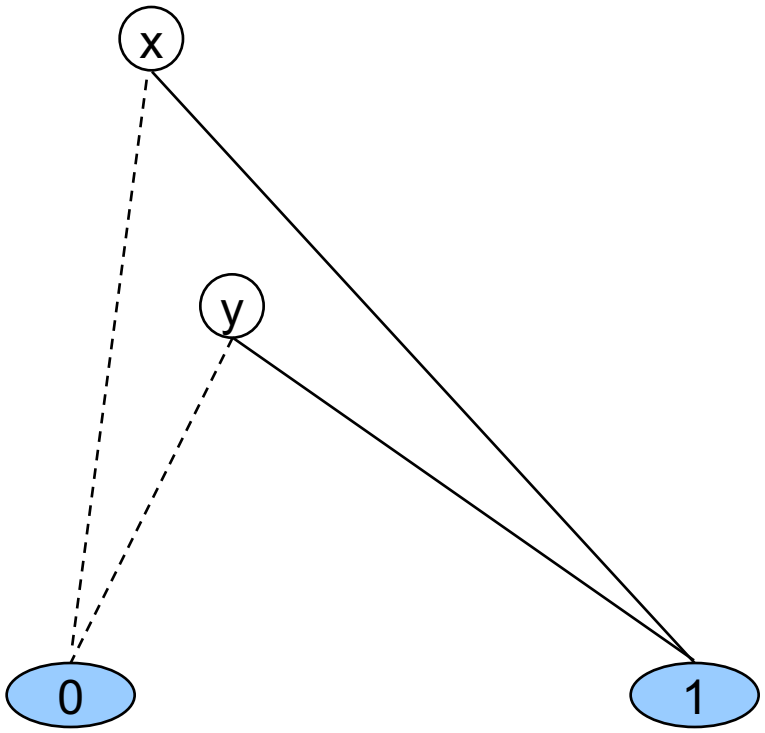
Bottom-up Compilation

CNF: (x + y) (y + z)

Variable order: x, y, z

Apply:

combines two OBDDs using Boolean operators



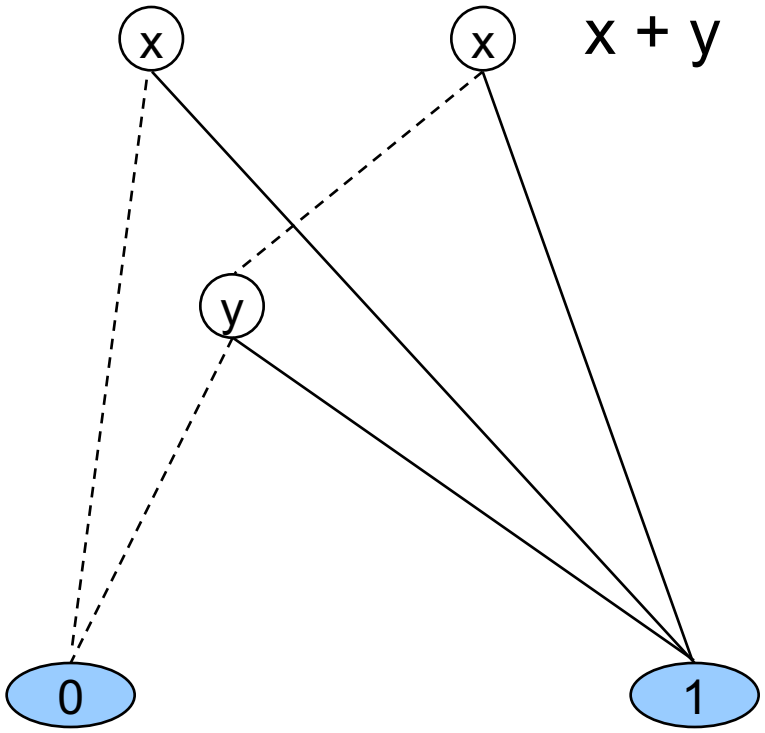
Bottom-up Compilation

CNF: (x + y) (y + z)

Variable order: x, y, z

Apply:

combines two OBDDs using Boolean operators



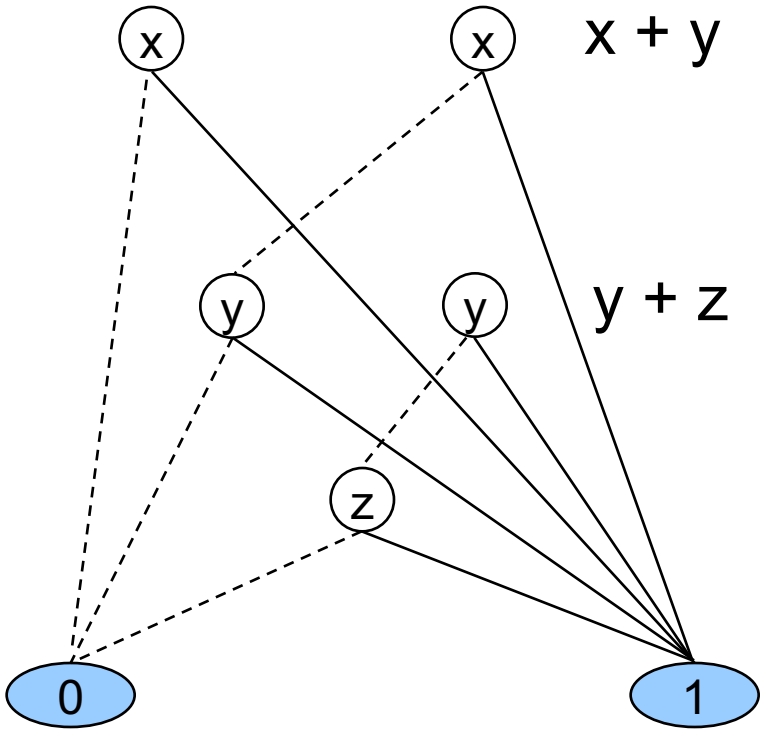
Bottom-up Compilation

CNF: (x + y) (y + z)

Variable order: x, y, z

Apply:

combines two OBDDs using Boolean operators



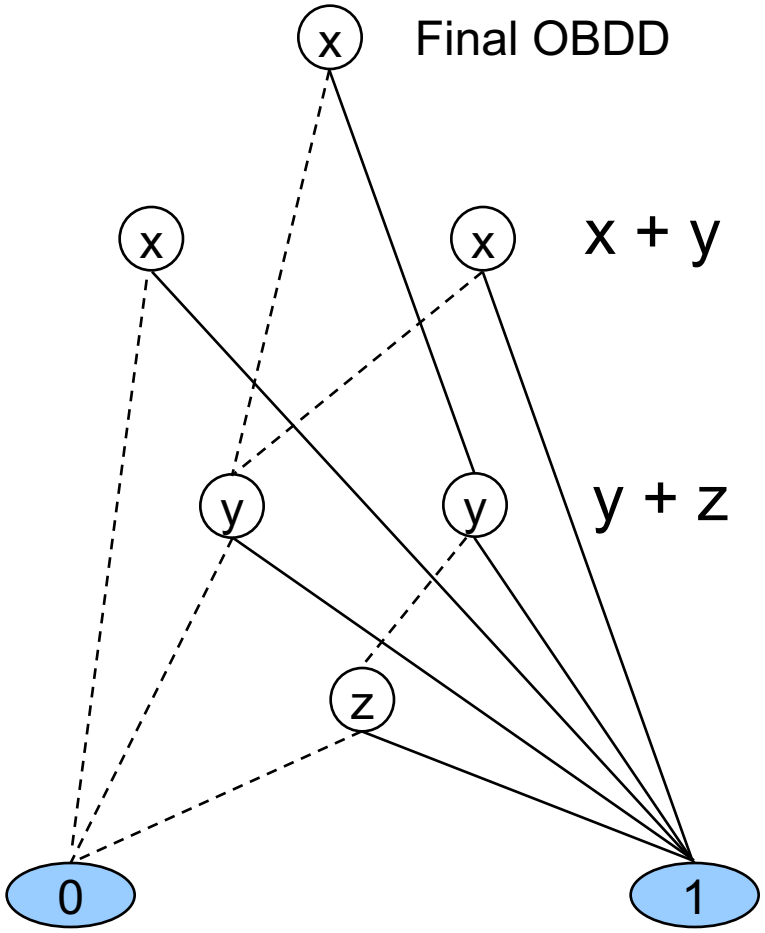
Bottom-up Compilation

CNF: (x + y) (y + z)

Variable order: x, y, z

Apply:

combines two OBDDs using Boolean operators



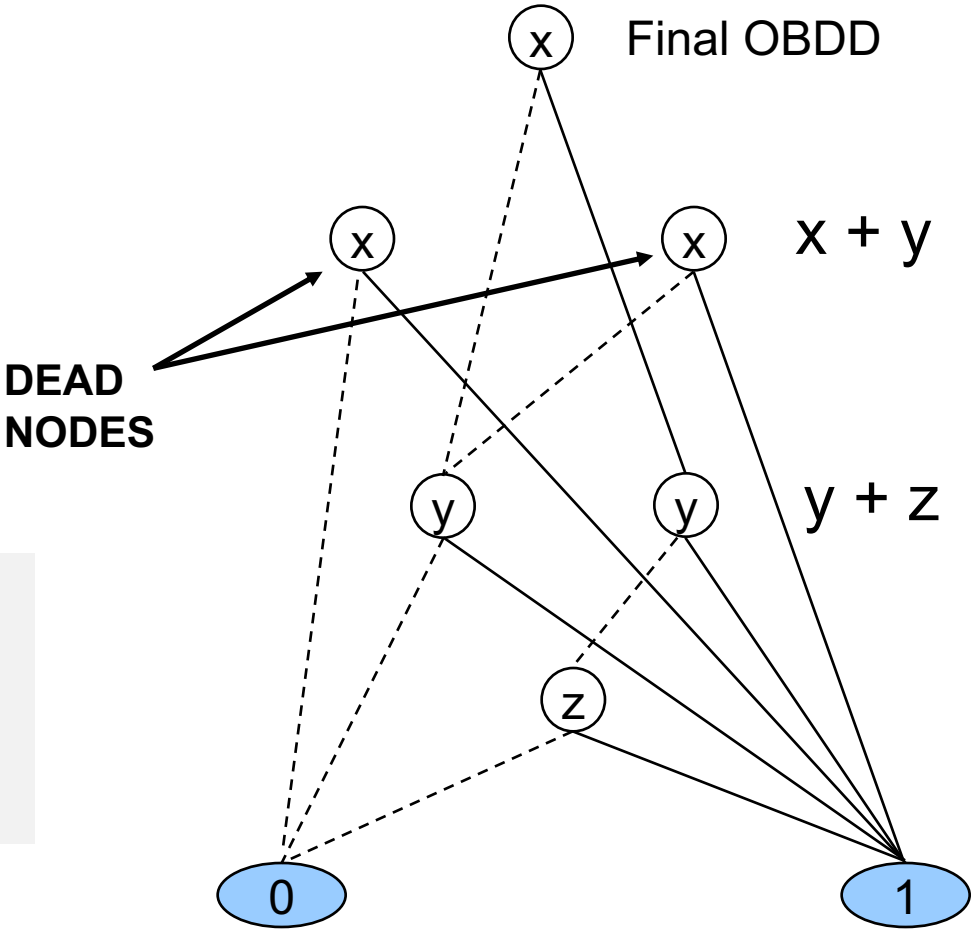
Bottom-up Compilation

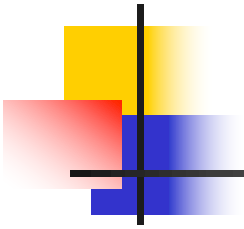
CNF: (x + y) (y + z)

Variable order: x, y, z

Apply:

combines two OBDDs using Boolean operators

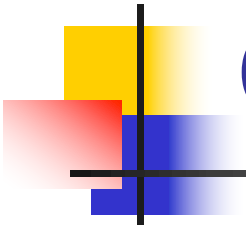




Bottom-up Compilation

- Requires:
  - *Apply* (conjoin, disjoin, etc)
  - Garbage collection of dead nodes
- Challenges:
  - Good variable order
  - Good schedule of Apply operations
- uf100-08 (32 models):
  - 176 nodes in final OBDD under MINCE variable order
  - 30,640,582 intermediate nodes using CUDD package

Canonicity in Compilation

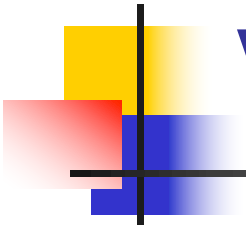


- OBDDs are canonical
  variable order → unique OBDD

  (reduced OBDDs)

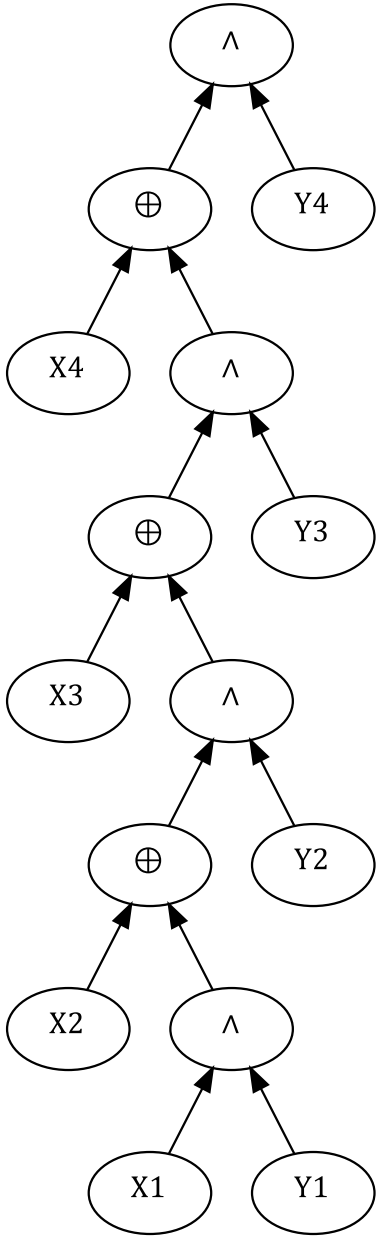
- SDDs are canonical
  vtree → unique SDD

  (trimmed and compressed SDDs)



Vtrees Matter!

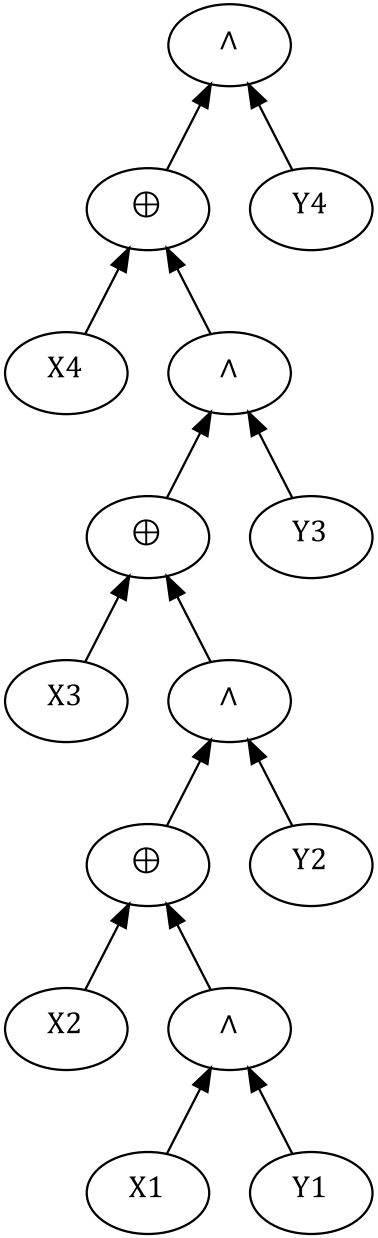
- A vtree can have a significant impact on the size of an SDD
- Good vtrees can be obtained either
  - **Statically**: by analyzing the Boolean function structure before compilation
  - **Dynamically**: by searching for an appropriate vtree during compilation



variables

$\mathbf{X} = \{ X_1\ X_2\ X_3\ \ldots\ X_n \}$

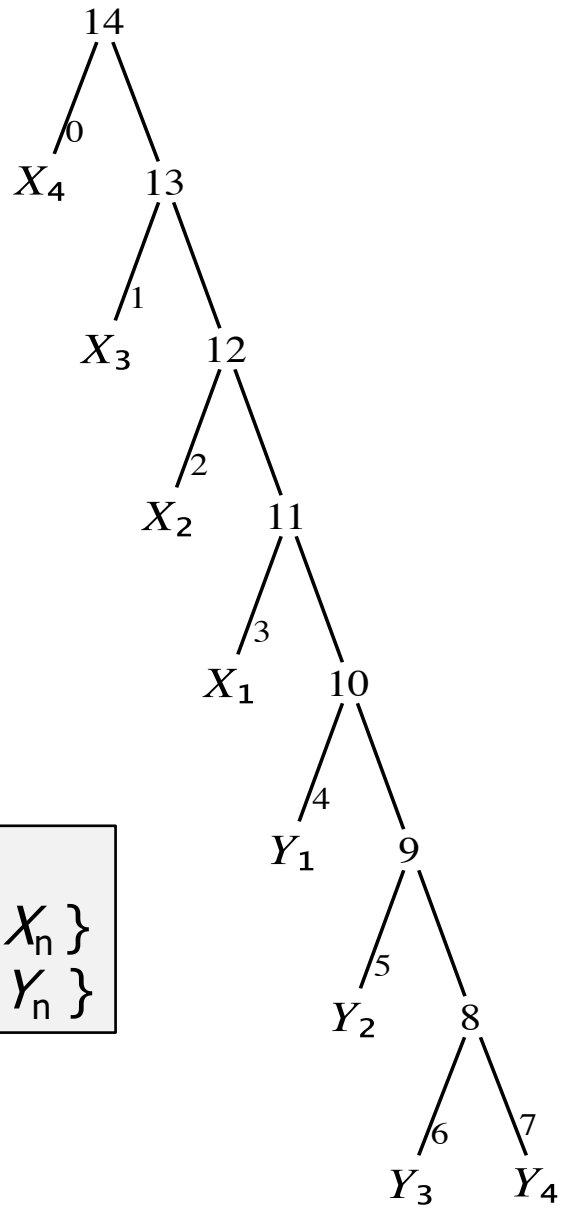
$\mathbf{Y} = \{ Y_1\ Y_2\ Y_3\ \ldots\ Y_n \}$

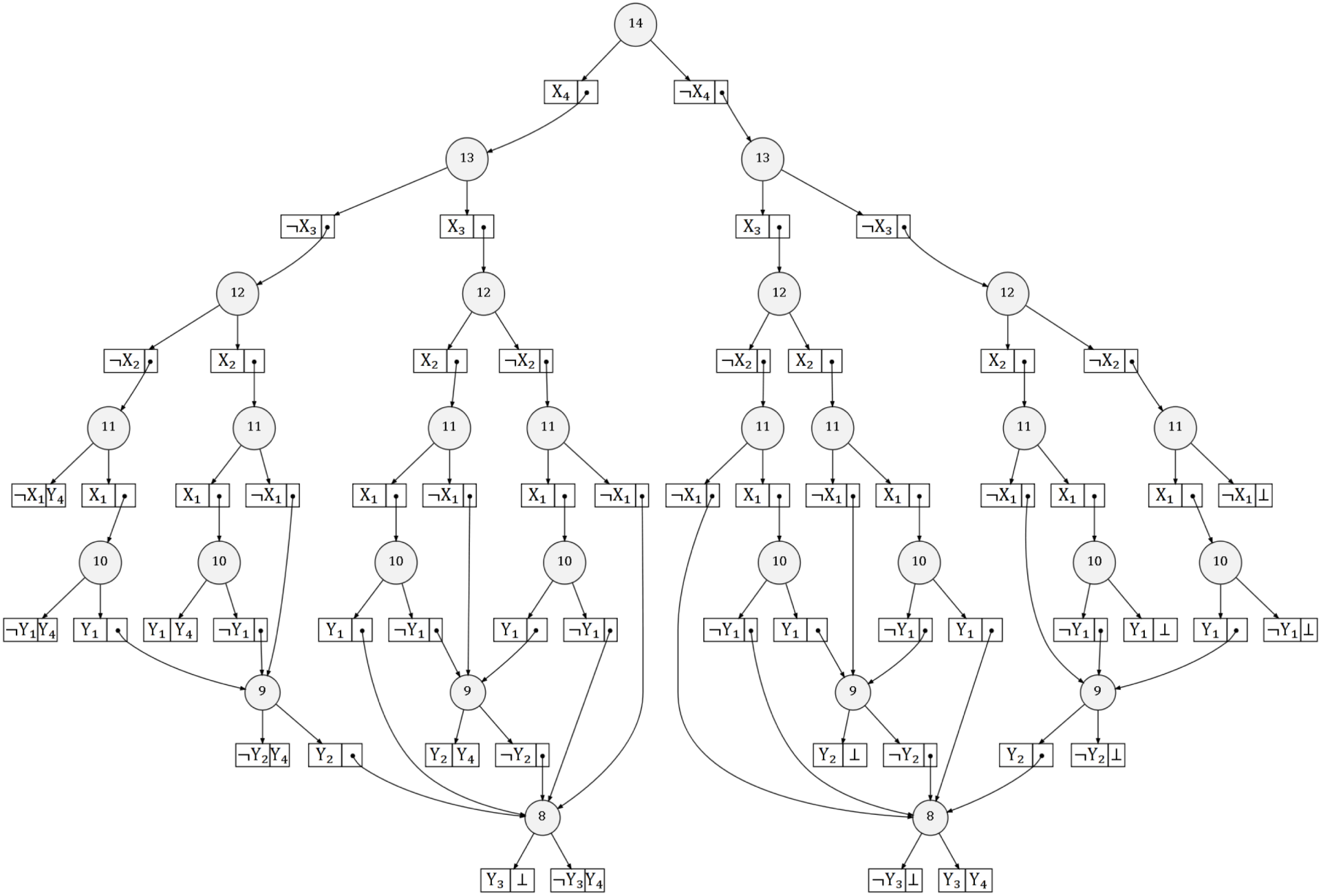


variables

X = { X_1 X_2 X_3 ... X_n }

Y = { Y_1 Y_2 Y_3 ... Y_n }




14
X_4
$\neg X_4$
13
13
$\neg X_3$
X_3
X_3
$\neg X_3$
12
12
12
12
$\neg X_2$
X_2
X_2
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X_2
X_2
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11
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11
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11
$\neg X_1$ Y_4
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Y_1
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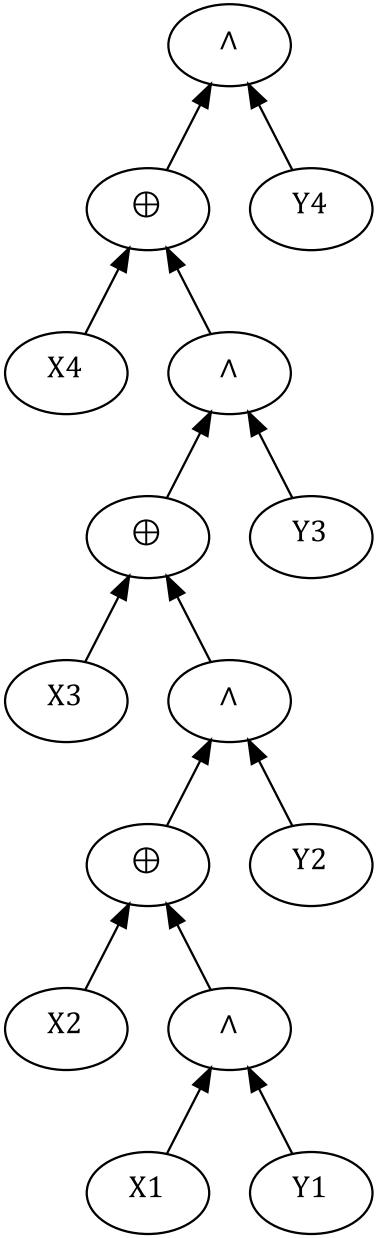
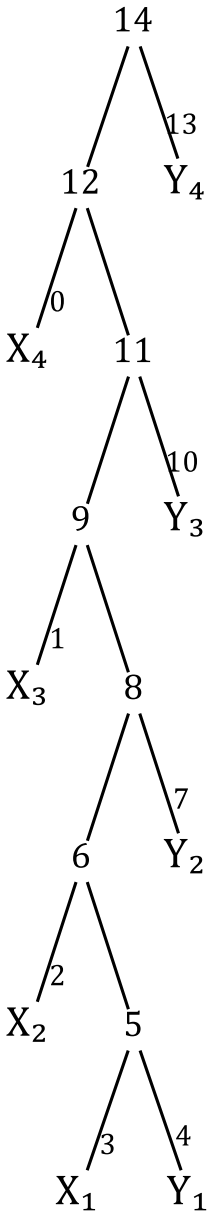
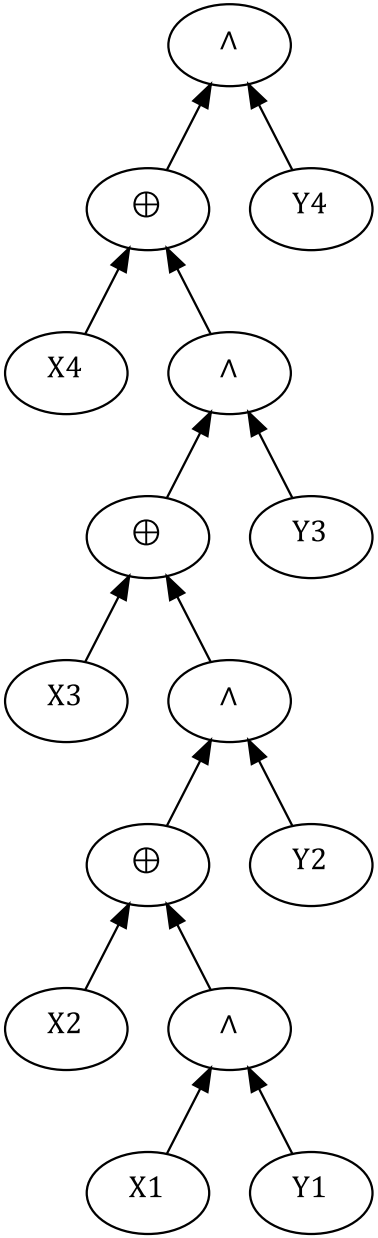
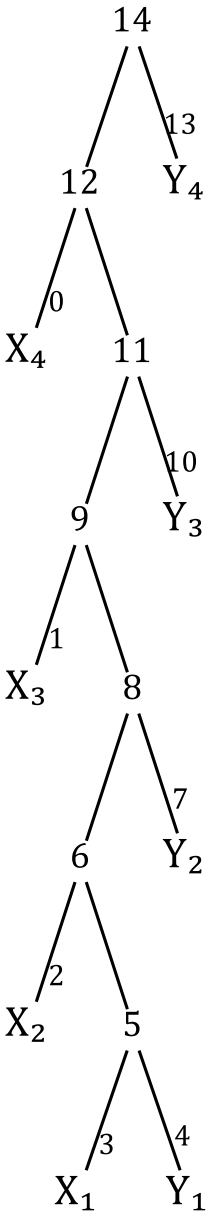
Y_2 $\bot$
$\neg Y_2$
Y_2
$\neg Y_2$ $\bot$
8
8
Y_3 $\bot$
$\neg Y_3$ Y_4
$\neg Y_3$ $\bot$
Y_3 Y_4


∧
⊕
Y4
X4
∧
⊕
Y3
X3
∧
⊕
Y2
X2
∧
X1
Y1


14
13
12
Y_4
0
X_4
11
10
9
Y_3
1
X_3
8
7
6
Y_2
2
X_2
5
3
4
X_1
Y_1


∧
⊕
Y4
X4
∧
⊕
Y3
X3
∧
⊕
Y2
X2
∧
X1
Y1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13
Y4
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X4
11
9
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Y3
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X3
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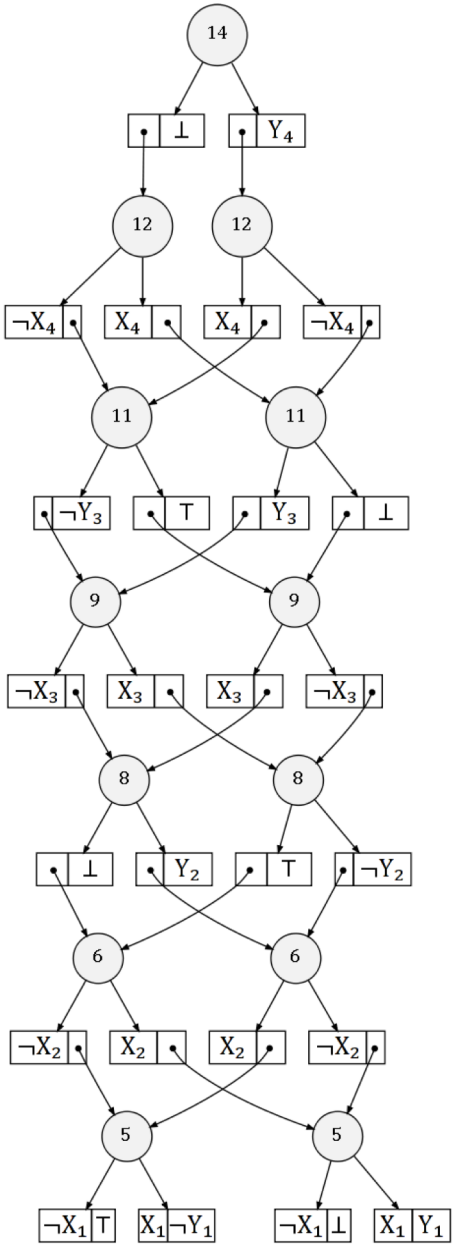
7
Y2
2
X2
5
3
4
X1
Y1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Y4
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X4
X4
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Y3
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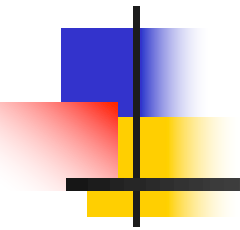
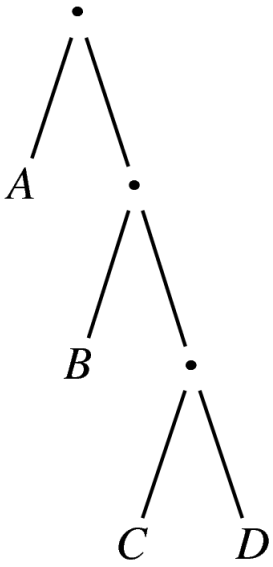
X1
Y1



Vtree Search: Minimizing SDD Size

Dissecting Variable Orders



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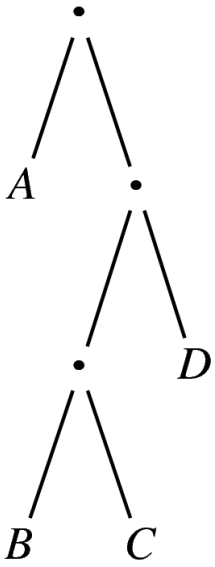

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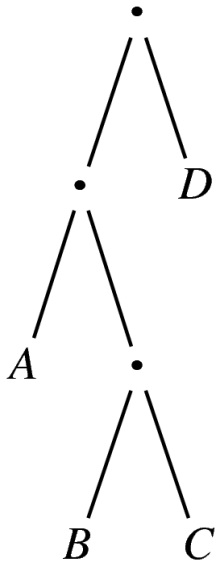

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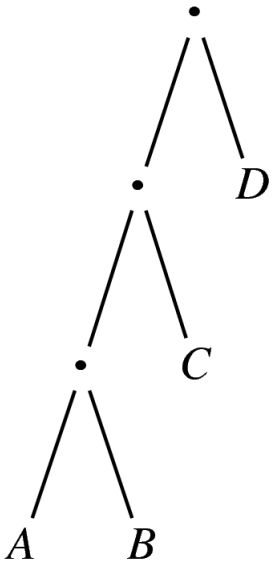

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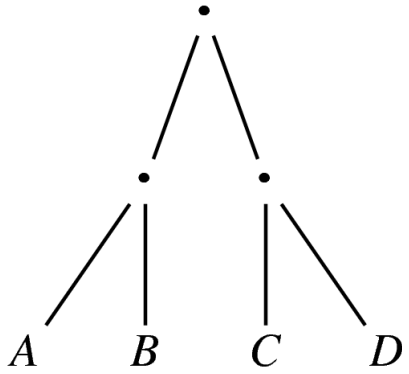

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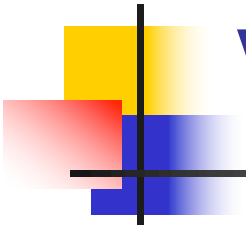
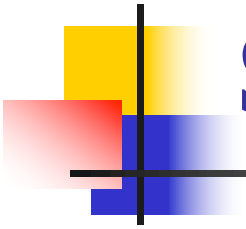


Vtrees Matter!

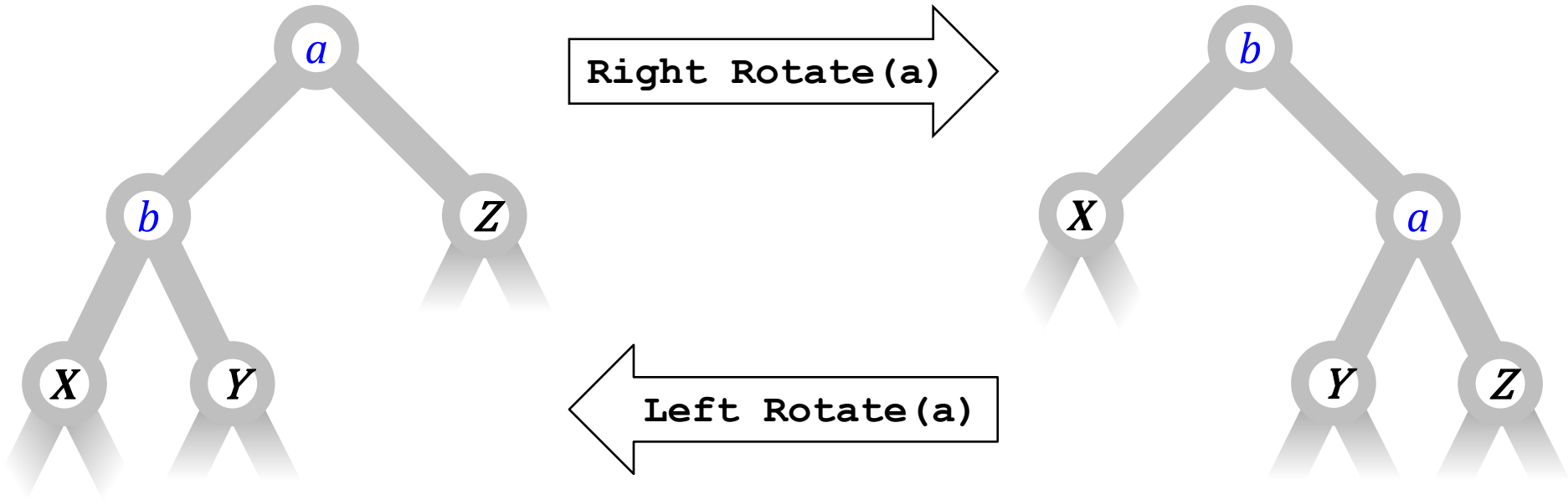
- The choice of a vtree can lead to exponential difference in the size of an SDD
- The choice of a dissection can also lead to exponential differences in the size of an SDD



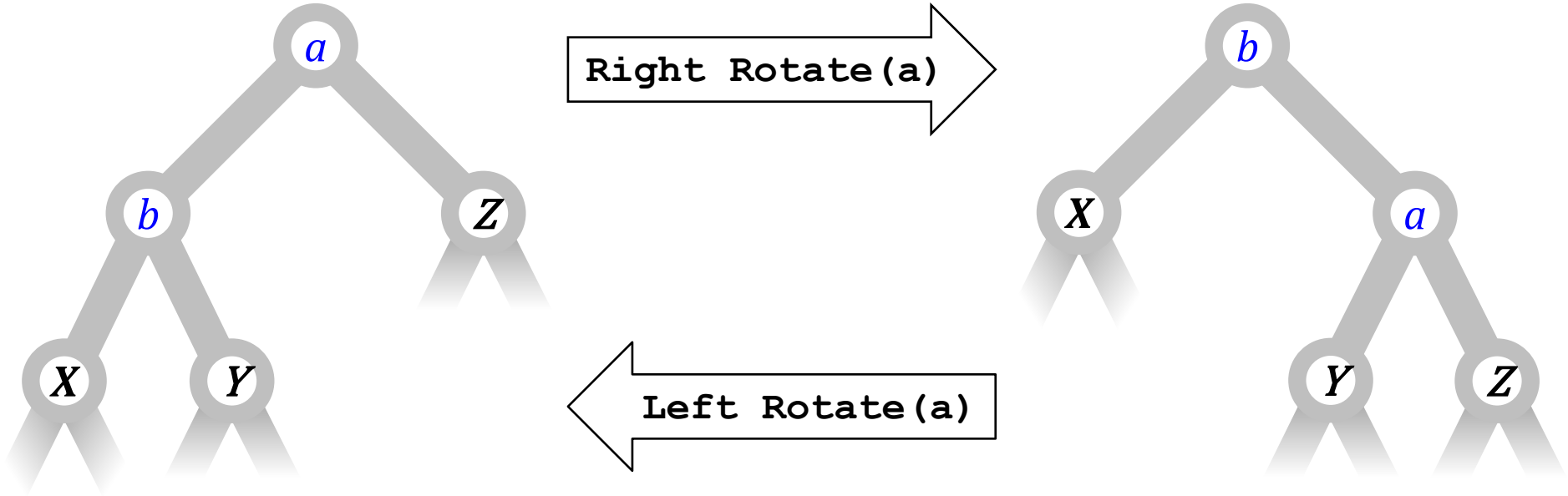
Searching Over Vtrees

- Double search problem:
  - Find variable order
  - Find dissection
- Tree operations:
  - Rotation (left, right)
  - Swapping
- Can enumerate all vtrees

Tree Rotations

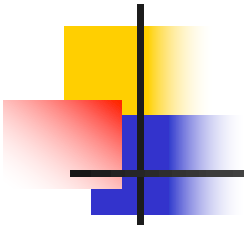


Rotation Preserves Order



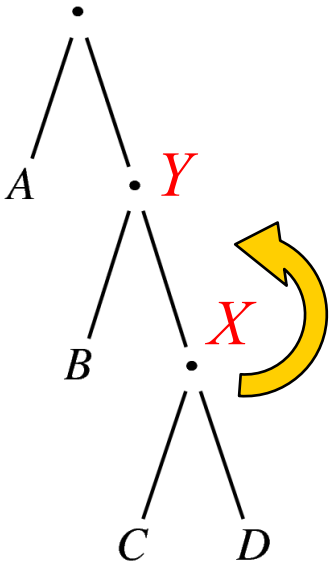
X, Y, Z

X, Y, Z



Enumerating Dissections

- Rotations can enumerate all dissection of a given variable order
- Systematic methods exist for this purpose
- See, e.g., Knuth's
  Art of Computer Programming, Volume 4, Fascicle 4: Generating All Trees



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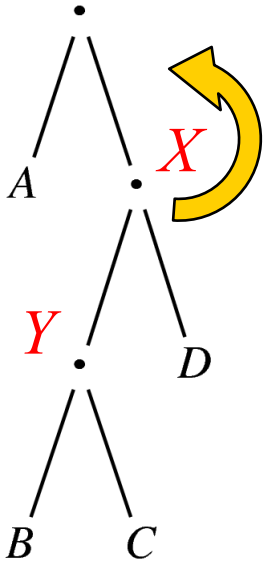

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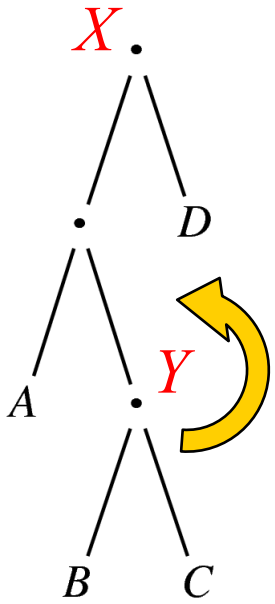

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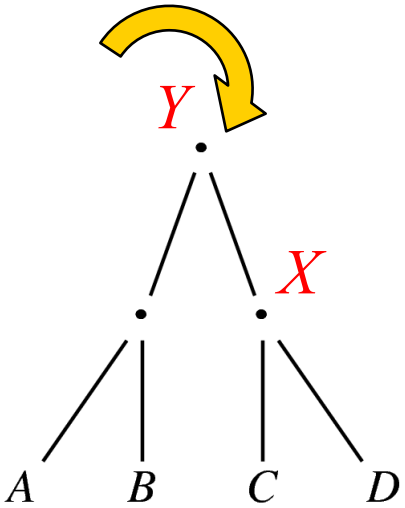

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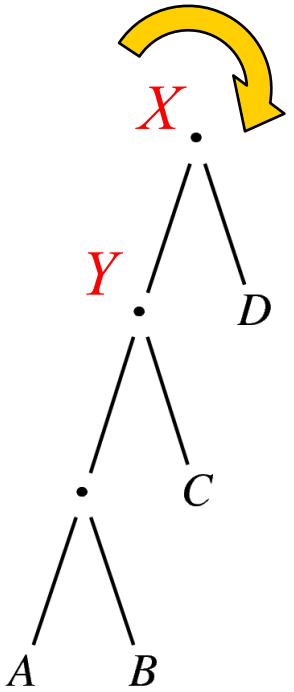
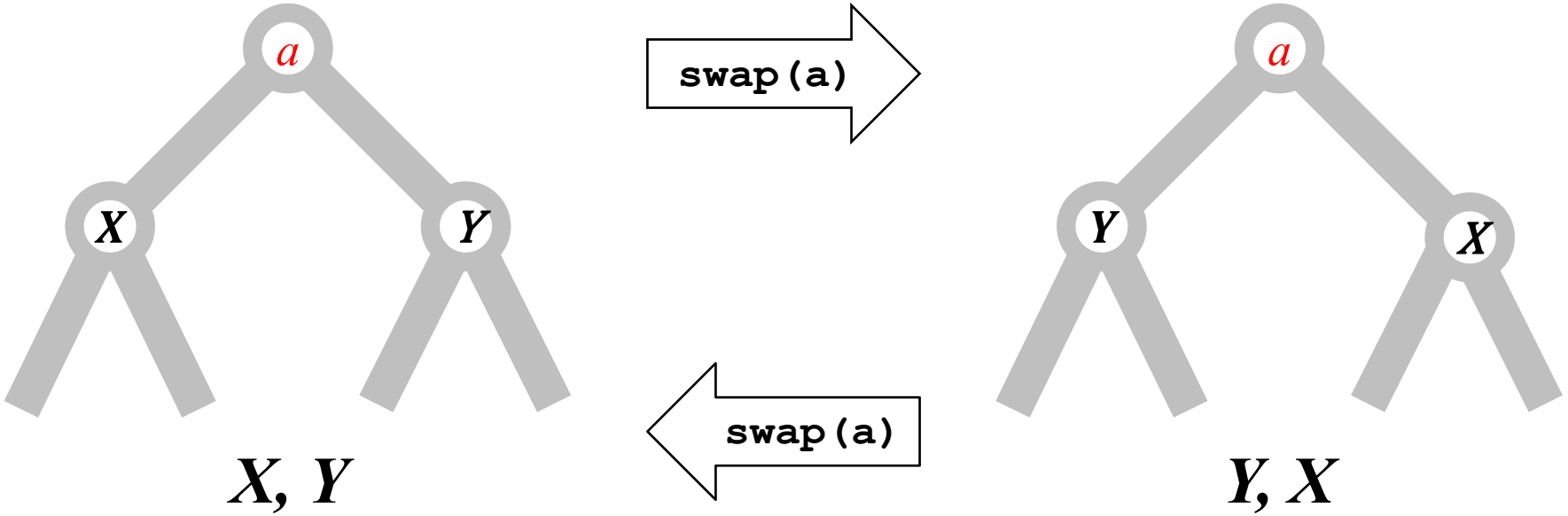
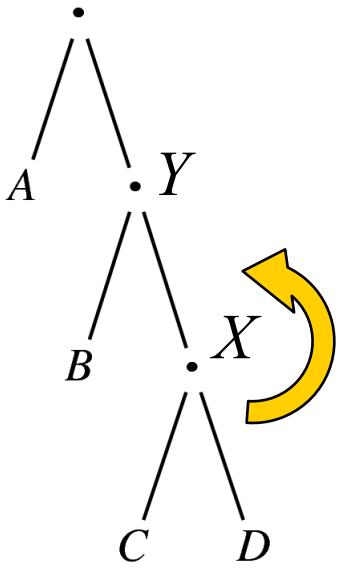


ABCD

Swapping Changes Order



Rotate + Swap



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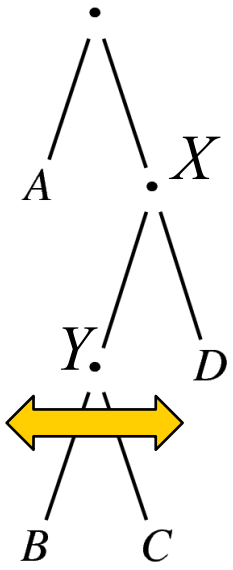

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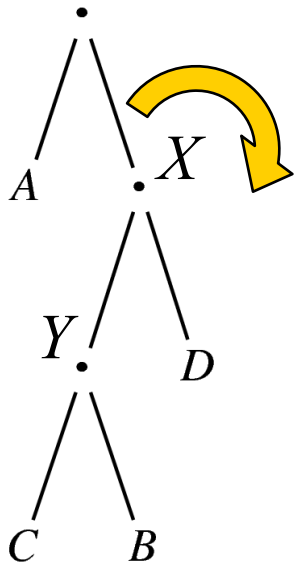

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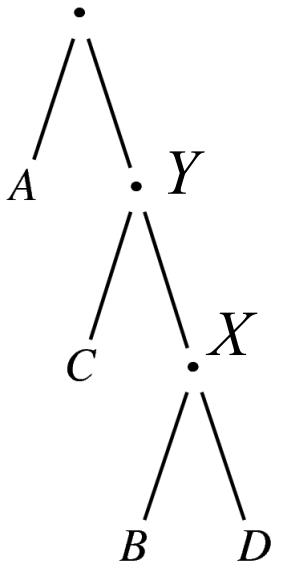
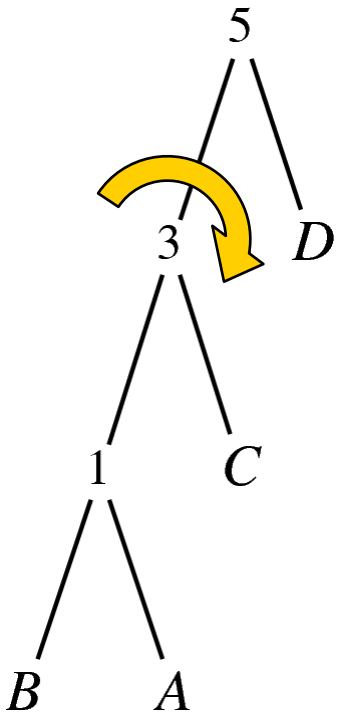
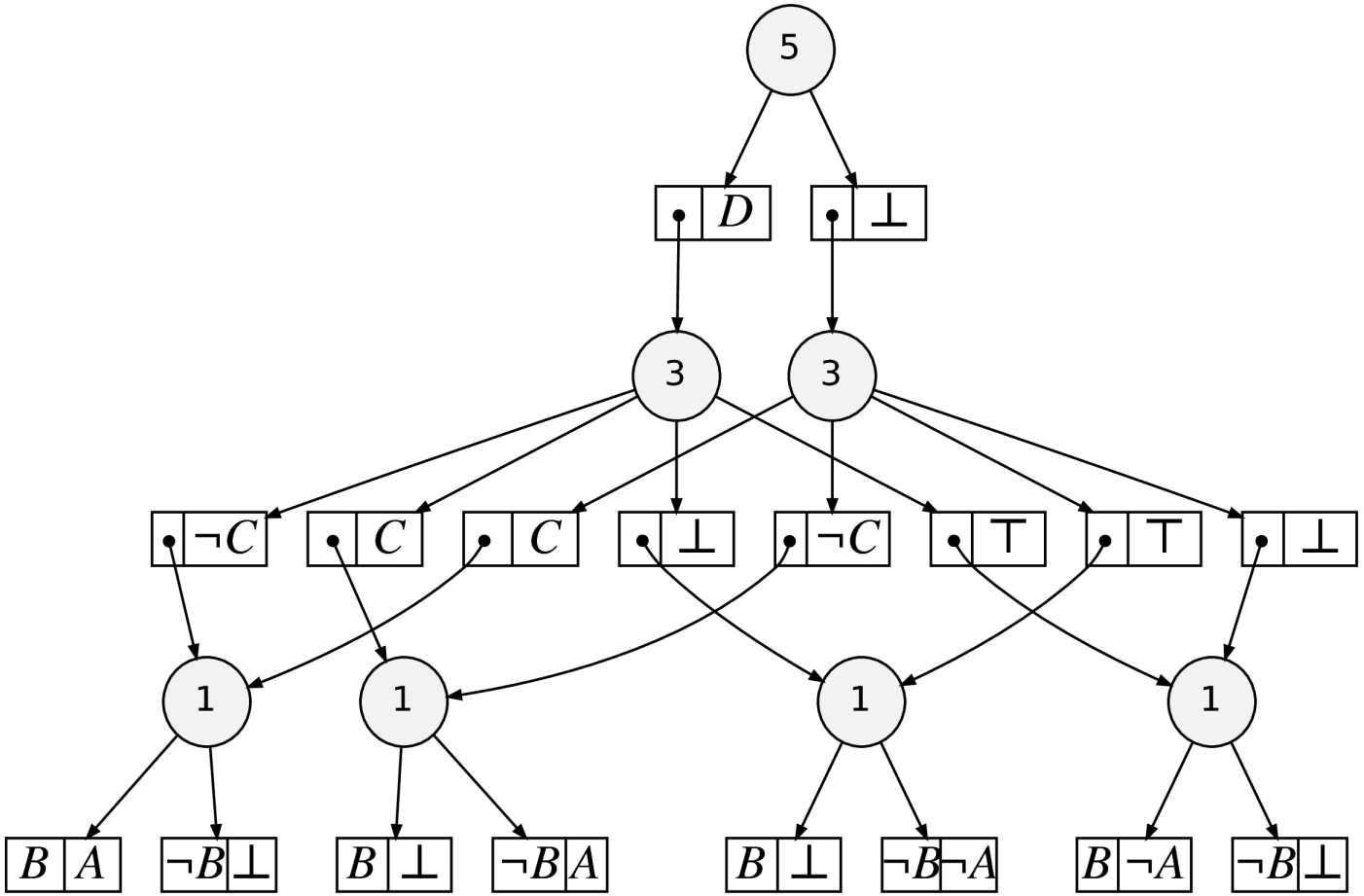


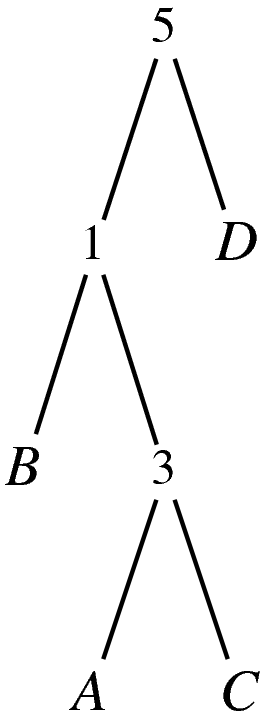
ACBD

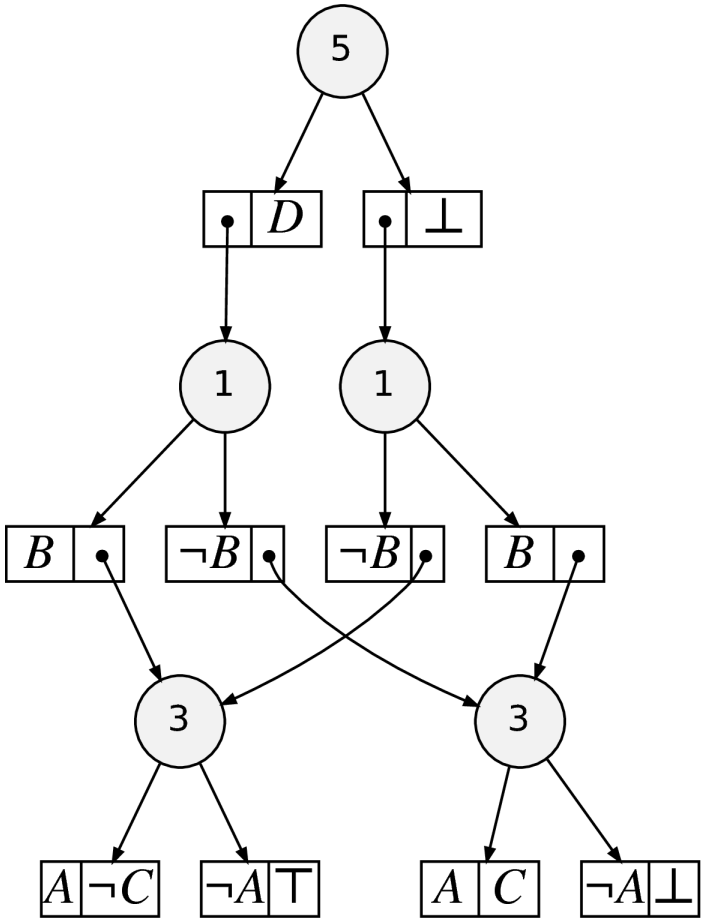
The SDD Package

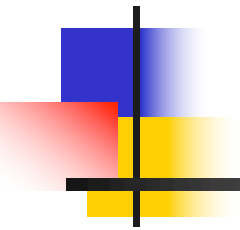




The SDD Package

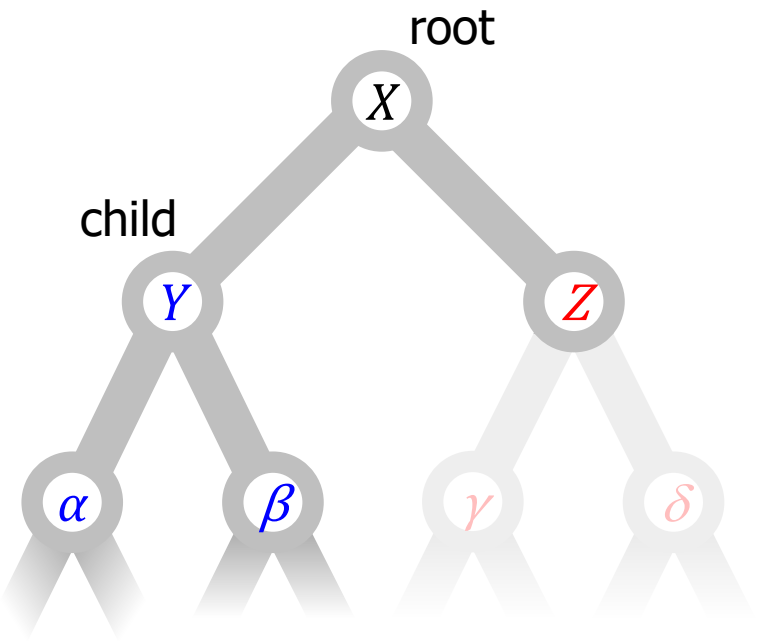






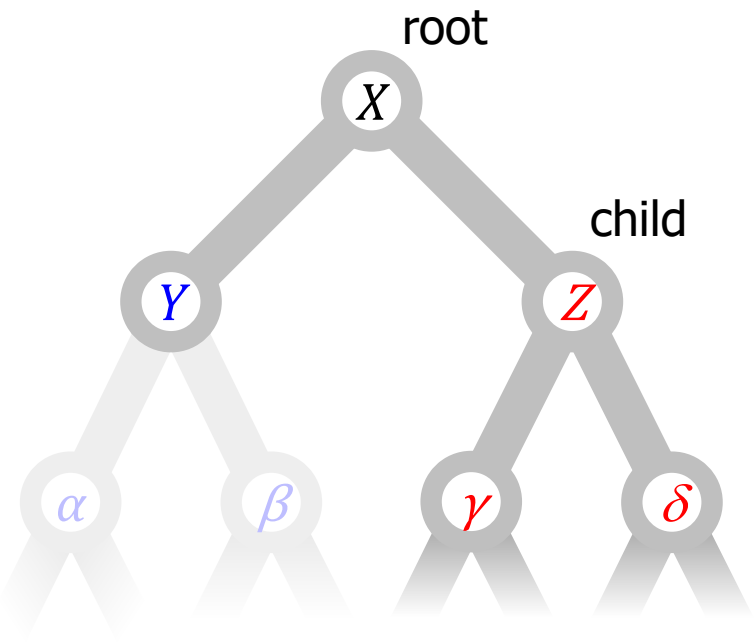
The Fragment Abstraction

Vtree Fragments

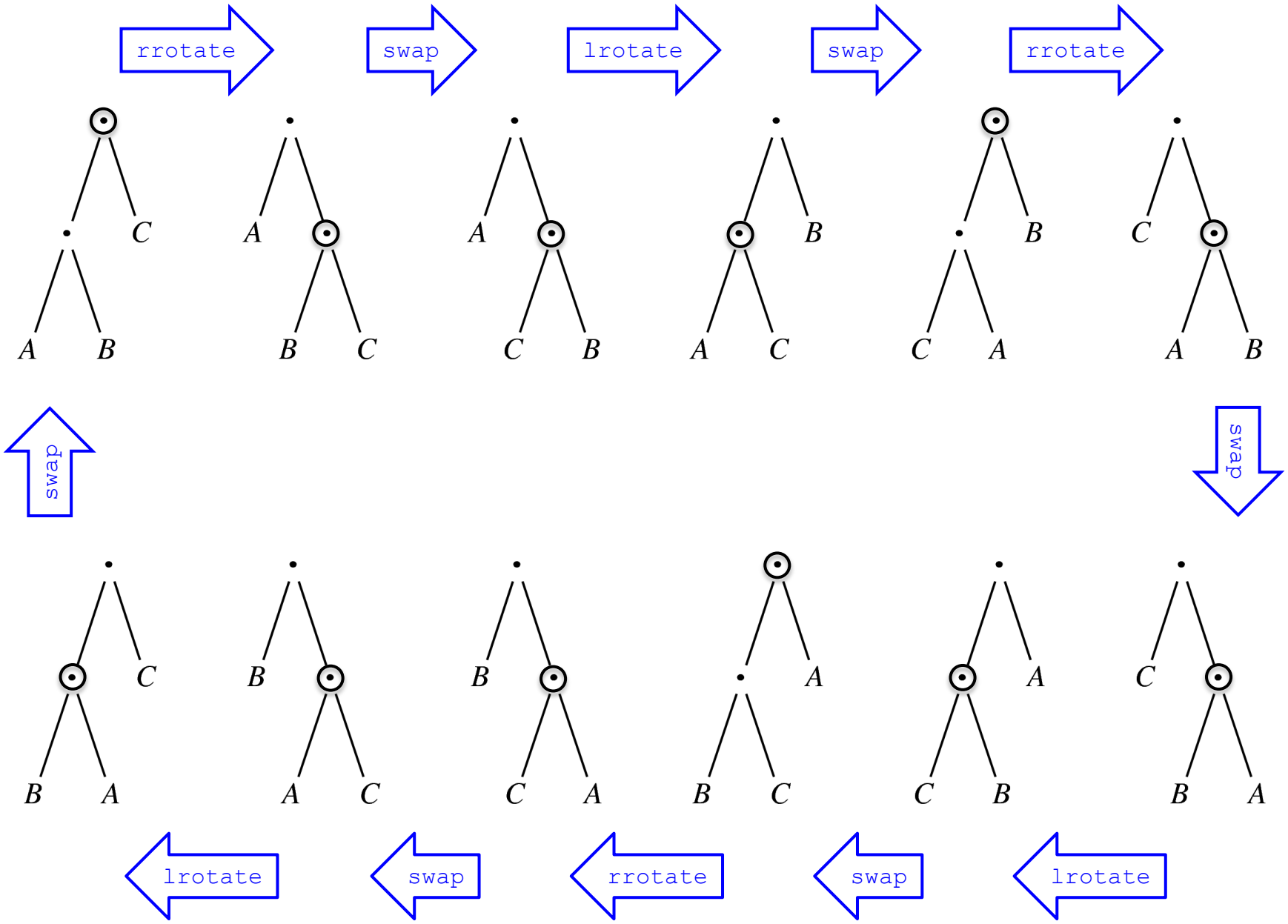


left-linear fragment

Vtree Fragments



right-linear fragment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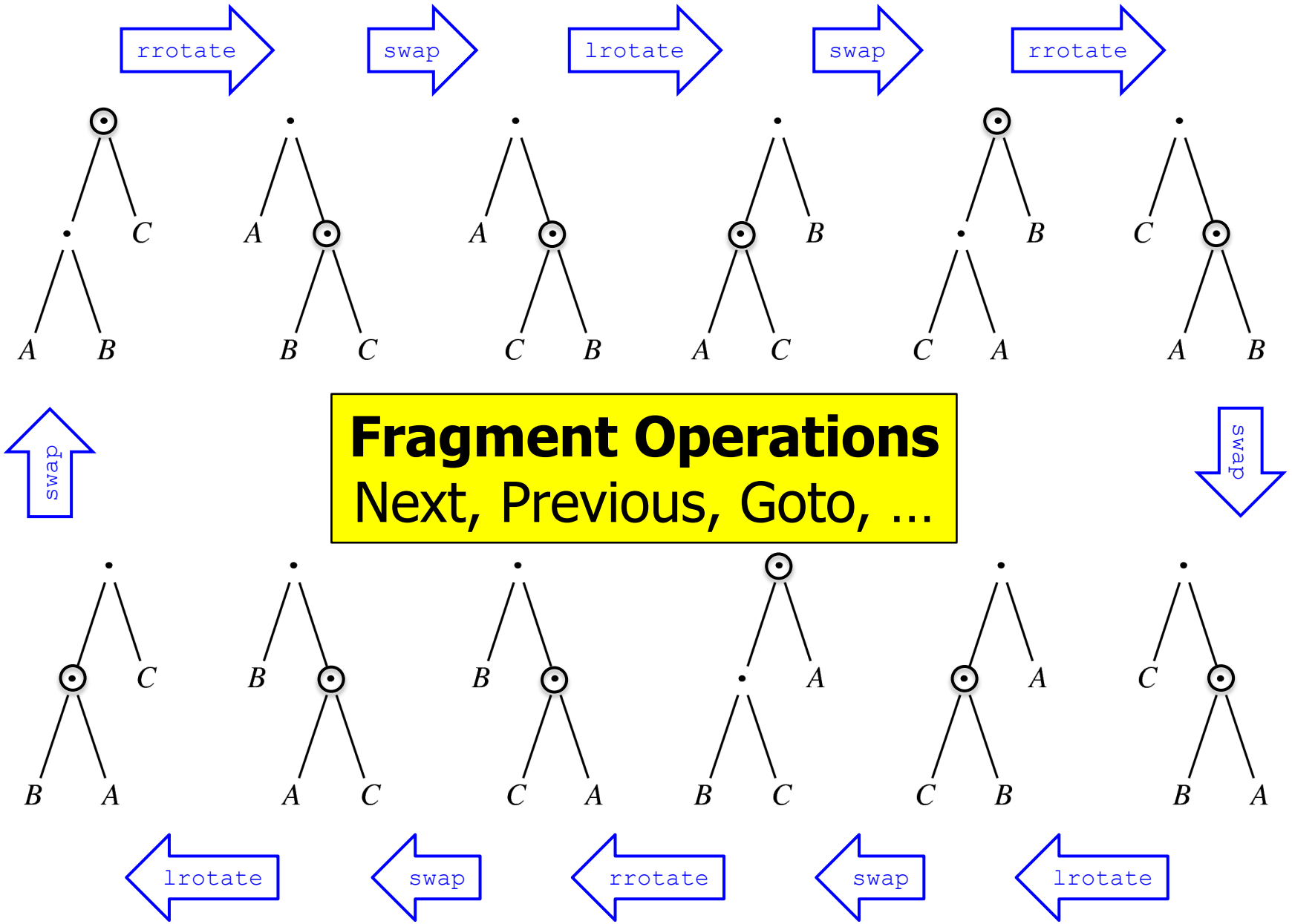
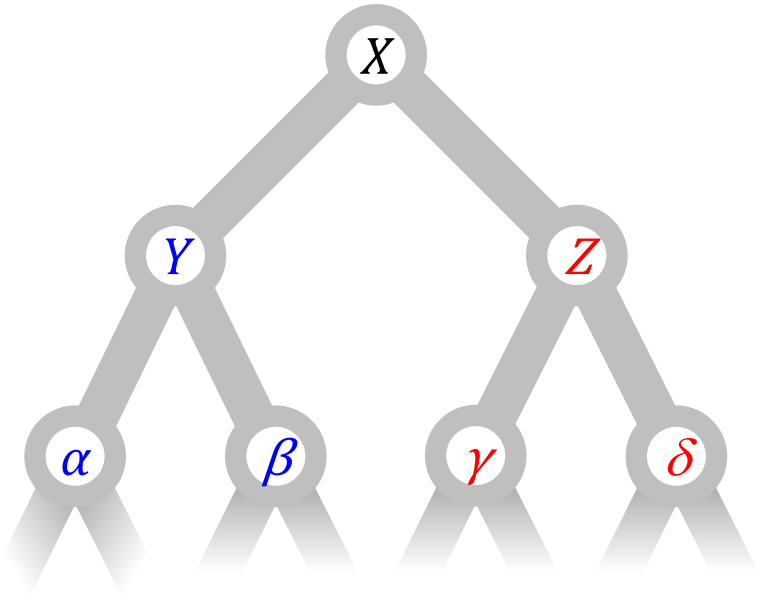
swap
lrotate
swap
rrotate
A
B
C
swap
swap
lrotate
swap
rrotate
swap
lrotate


rrotate
swap
lrotate
swap
rrotate
A
B
C
swap
swap
Fragment Operations
Next, Previous, Goto, ...
lrotate
swap
rrotate
swap
lrotate

Greedy Vtree Search

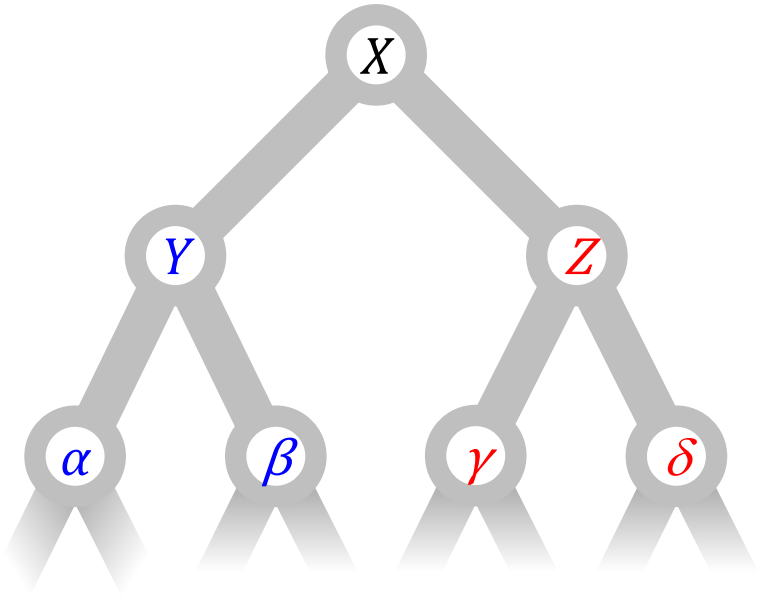
- Traverse vtree bottom-up
- Try enumerating 12 vtrees of left-linear fragment
- Try enumerating 12 vtrees of right-linear fragment
- 24 vtrees in total
- Greedily accept best vtree
- Prune parts of search space

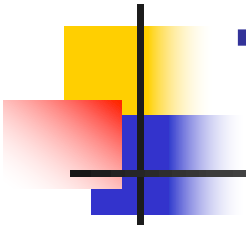


The SDD Package

Opportunities

- vtree search algorithms
- Triggers for vtree search
- Static methods for constructing vtrees
- Heuristics for scheduling Apply operations





The SDD Package

- http://reasoning.cs.ucla.edu/sdd
- Written in C
- Available as open-source
- Code for compiling CNFs/DNFs into SDDs (includes heuristic for scheduling Apply)
- Two manuals: beginner and advanced

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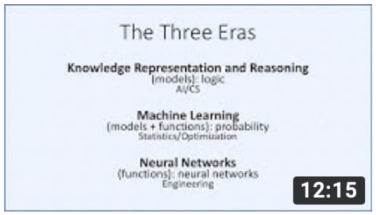

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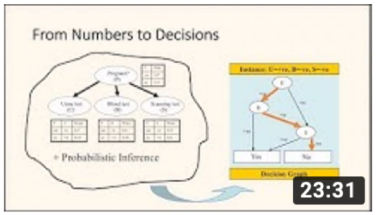

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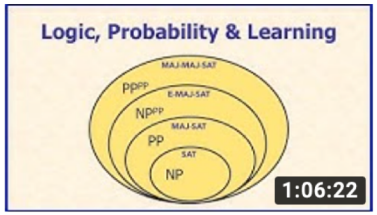

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