On Finding Minimum Satisfying Assignments

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Problem definition

Minimum satisfying assignment

what is a *minimum satisfying assignment* (MSA)?

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$$x + y + z > 0 \quad \vee \quad w + x + y + z < 5$$

$$w, x, y, z \in \mathbb{Z}$$

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$w = 2 \quad x = 1 \quad y = 0 \quad z = 0$ — *SA*

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⋆	~~$w = 2$~~	$x = 1$	$y = 0$	$z = 0$	— *minimal SA (mSA)*

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$$(x + y + z > 0 \quad \vee \quad x + y + z < 3)$$

given $\mathbb{U} = \{a, b, c, d, e, f, g, h\}$ and its subsets

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	$\{b, e, f, g\}$	— *HS (hitting set)*
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⋆	$\{a, d, g, \cancel{h}\}$	— *minimal HS (mHS)*
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given F s.t. $\mathrm{var}(F) = X \cup Y$,

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$$\exists_X \forall_Y .\, F\ ?$$

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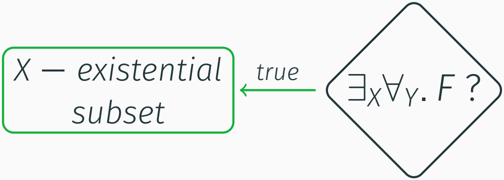

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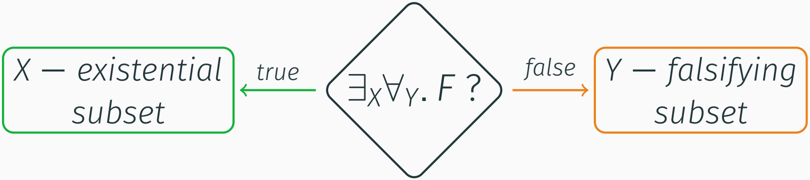

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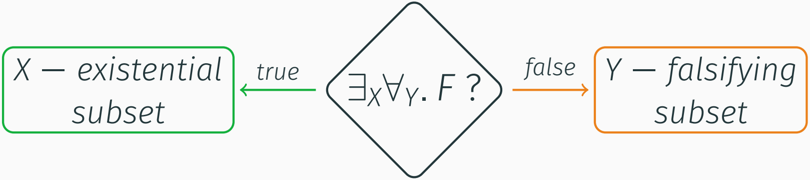

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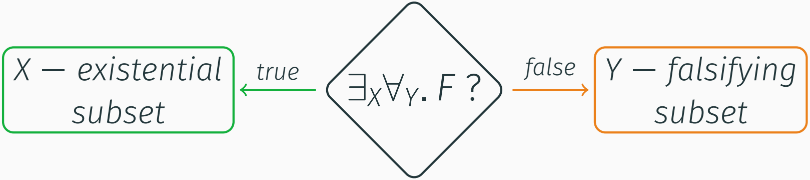

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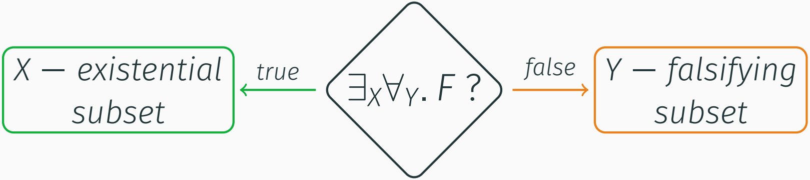

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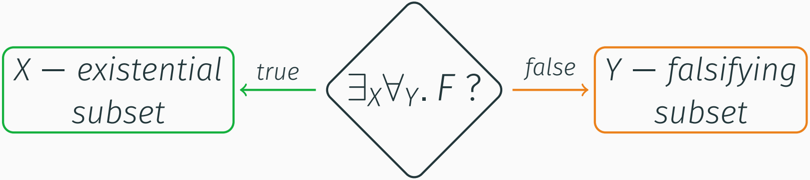

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	$\{w, x, y, z\} \cup \emptyset$	$\exists_{w,x,y,z}.\, F = \mathit{true}$	$\{w, x, y, z\}$	— *existential set (ES)*

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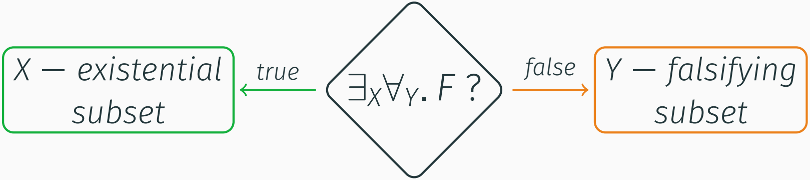

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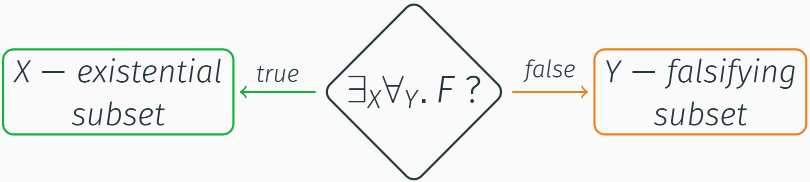

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Approach

Minimal hitting set duality

given F,

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$\mathcal{E}$ — set of **all mESes** for F

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1. set $e \in \mathcal{E}$

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Algorithm

input : formula F
output: an MSA of F

```
𝓗 ← ∅
while true:
    X ← MinHS(𝓗)                      # get a new MHS with MaxSAT
    Y ← var(F) \ X                     # take complement of X
    (st, μ_X) ← Solve(∃_X ∀_Y. F)      # check if X is a minimum ES
    if st:
        break
    else:
        I ← Reduce(Y)                  # reduce counterexample
        𝓗 ← 𝓗 ∪ I                      # hit counterexample next time
return MSA ← μ_X
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$\mathcal{H} \leftarrow \emptyset$
while true:
    $X \leftarrow$ `MinHS`$(\mathcal{H})$ # get a new MHS with MaxSAT
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        ℋ ← ℋ ∪ Y                               # hit counterexample next time
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Example

$$F = ((a + b \geqslant 0) \vee (c \leqslant 0)) \wedge ((a + b \geqslant 0) \vee (b - a \leqslant 0))$$

$$\mathrm{var}(F) = \{a, b, c\}$$

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$X \leftarrow \{b\}$	*false*	$I \leftarrow \{a\}$	

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MinHS($\mathcal{H}$)	Solve($\exists_X \forall_Y . F$)	$I \leftarrow$ Reduce(Y)	$\mathcal{H} = \mathcal{H} \cup I$
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$X \leftarrow \{\emptyset\}$	*false*	$I \leftarrow \{b, c\}$	$\{\{b, c\}\}$
$X \leftarrow \{b\}$	*false*	$I \leftarrow \{a\}$	$\{\{b, c\}, \{a\}\}$

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$X \leftarrow \{\emptyset\}$	*false*	$I \leftarrow \{b, c\}$	$\{\{b, c\}\}$
$X \leftarrow \{b\}$	*false*	$I \leftarrow \{a\}$	$\{\{b, c\}, \{a\}\}$
$X \leftarrow \{a, c\}$	*true*	$\{a = 1, c = 0\}$ is an MSA of F	

$$((b + 1 \geqslant 0) \vee (0 \leqslant 0)) \wedge ((b + 1 \geqslant 0) \vee (b - 1 \leqslant 0))$$

Experimental results

Experimental evaluation

- new IHS-based approach:
  1. MINT
  2. MINT+ $=$ MINT $+$ bootstrapping with unit sets:

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- minimum hitting set engine — incremental MaxSAT

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- Machine configuration:
  - Intel Xeon E5-2630 2.60GHz with 64GByte RAM

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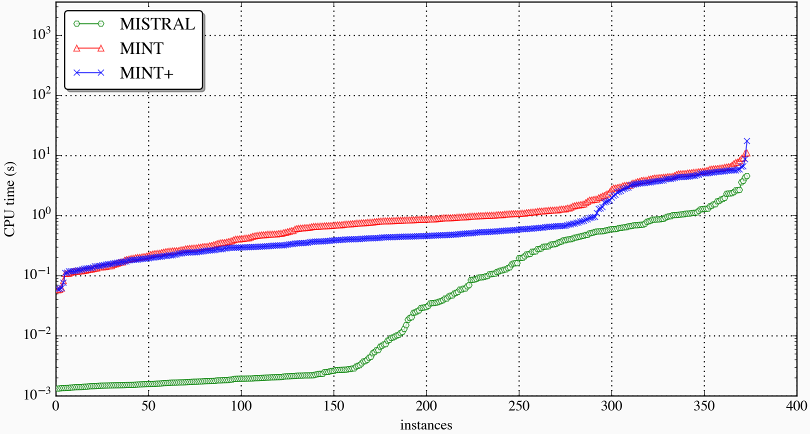
Original CAV12 benchmarks



too easy!

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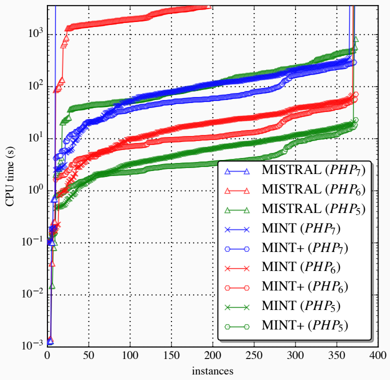
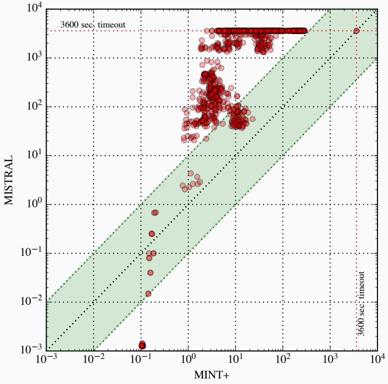


(a) cactus plot



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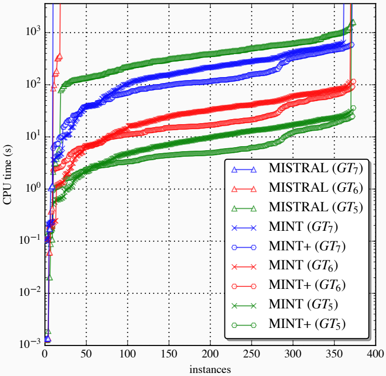

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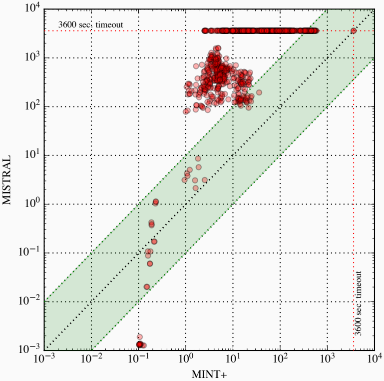

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Performance of MINT+ vs. MISTRAL

	PHP_5	PHP_6	PHP_7
MINT+	373 *(4.74s)*	**373** ***(15.3s)***	**371** ***(>84.0s)***
MISTRAL	373 *(149.4s)*	195 *(>2784.1s)*	9 *(>3513.2s)*

	GT_5	GT_6	GT_7
MINT+	373 *(6.9s)*	**373** ***(24.0s)***	**371** ***(>165.2s)***
MISTRAL	373 *(421.9s)*	18 *(>3431.8s)*	9 *(>3474.6s)*

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- better IHS-based algorithms
- more practical applications of MSA

Questions?