Maximal Falsifiability: Definitions, Algorithms, and Applications

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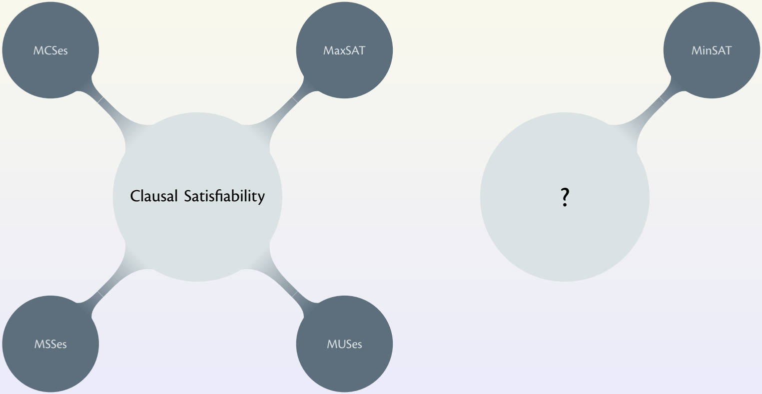
Motivation



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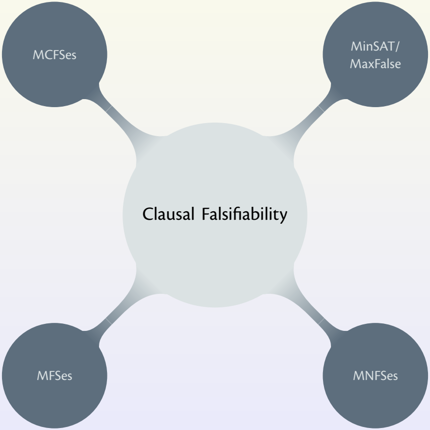
MCFSes
MinSAT/
MaxFalse
Clausal Falsifiability
MFSes
MNFSes

Maximum Falsifiability

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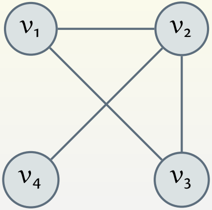
$\mathcal{F} = \mathcal{H} \cup \mathcal{R}$
$\mathcal{H} = \{ x \vee y \vee z \}$
$\mathcal{R} = \{ x, \quad y, \quad z \}$

Only one MNFS $\mathcal{N} = \mathcal{R}, \quad |\mathcal{N}| = 3$.

Connection to Maximal Independent Set



(a) Graph $\mathcal{G}$

$$\mathcal{F} = \left\{ \begin{array}{l} c_1 = x_{v_1 v_2} \vee x_{v_1 v_3} \\ c_2 = \neg x_{v_1 v_2} \vee x_{v_2 v_3} \vee x_{v_2 v_4} \\ c_3 = \neg x_{v_1 v_3} \vee \neg x_{v_2 v_3} \\ c_4 = \neg x_{v_2 v_4} \end{array} \right\}$$

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Figure : From maximal independent set to maximal falsifiability

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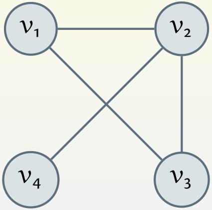

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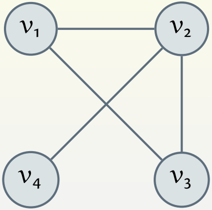

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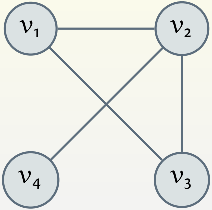

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Let $\mathbb{N}(\mathcal{F})$ and $\mathbb{C}(\mathcal{F})$ be complete sets of MNFSes and MCFSes of $\mathcal{F}$, respectively.

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foreach c ∈ R:
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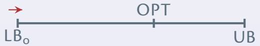

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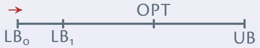

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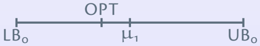

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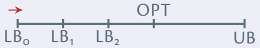
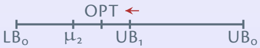


linear search SAT/UNSAT

binary search



Iterative MaxFalse algorithms

- similar to iterative MaxSAT
- SAT solver is used as an oracle
- consider formula $\mathcal{F} = \mathcal{H} \cup \mathcal{R}$
- $\forall c_i \in \mathcal{R}$ modify $\mathcal{H}$ and $\mathcal{R}$:
  - $\mathcal{H} \leftarrow \mathcal{H} \cup \{\neg l_{i_j} \vee r_i\} \ \forall l_{i_j} \in c_i$
  - $\mathcal{R} \leftarrow \mathcal{R} \setminus \{c_i\}$
- relaxation constraints $\sum_{r_i} r_i \leqslant k$
- varying k using 3 algorithms:

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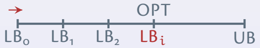

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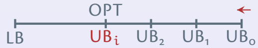
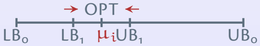
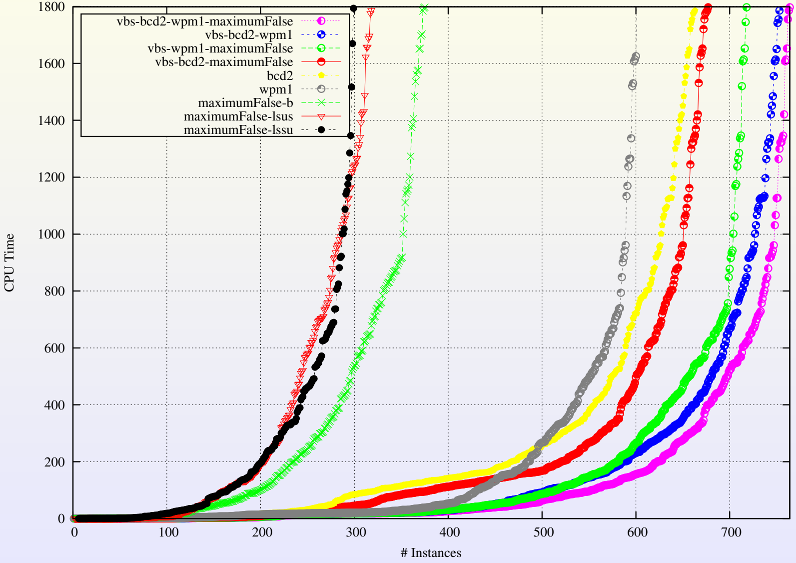


binary search



Performance comparison: MaxFalse for MaxSAT instances



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- More practical applications

Thank you for your attention!