Cardinality Encodings for Graph Optimization Problems

Alexey Ignatiev[1,2], Antonio Morgado[1], and Joao Marques-Silva[1]

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[1] LASIGE, FC, University of Lisbon, Portugal
[2] ISDCT SB RAS, Irkutsk, Russia

Definitions

given a *propositional* formula in CNF,

given a ***propositional*** formula in CNF,
decide whether or not it is satisfiable

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Example

$\mathcal{F}_1 = (x_1) \wedge (\neg x_1 \vee \neg x_2)$

given a *propositional* formula in CNF, decide whether or not it is satisfiable

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$\mathcal{F}_1 = (x_1) \wedge (\neg x_1 \vee \neg x_2)$ – satisfiable

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$\mathcal{F}_1 = (x_1) \wedge (\neg x_1 \vee \neg x_2)$ — satisfiable

$\mathcal{F}_2 = (x_1) \wedge (\neg x_1 \vee \neg x_2) \wedge (x_2)$

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Example

$\mathcal{F}_1 = (x_1) \land (\neg x_1 \lor \neg x_2)$ – satisfiable

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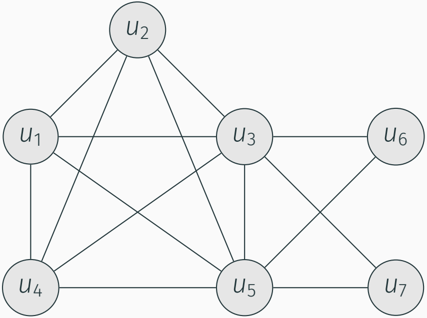
Graph problems

graph $G = (V, E)$:



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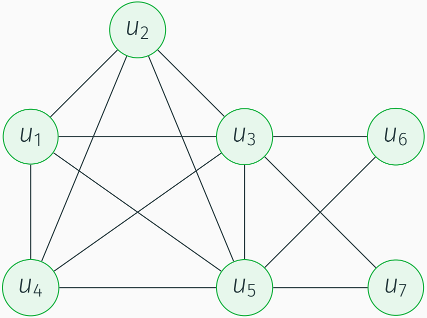

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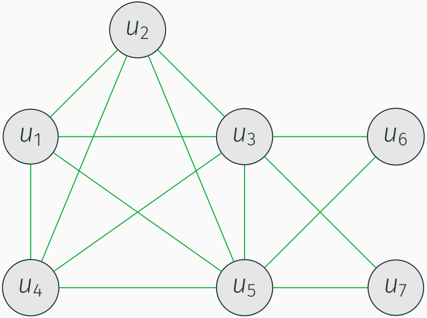
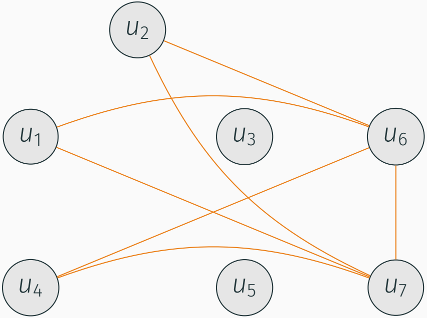


Graph problems

graph $G^C = (V, E^C)$:



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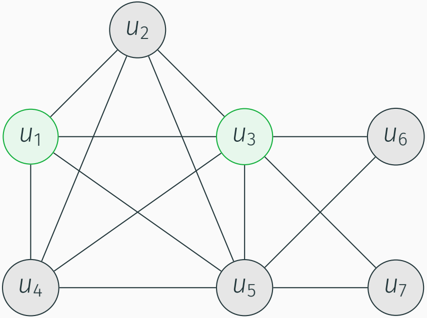

clique of size 2

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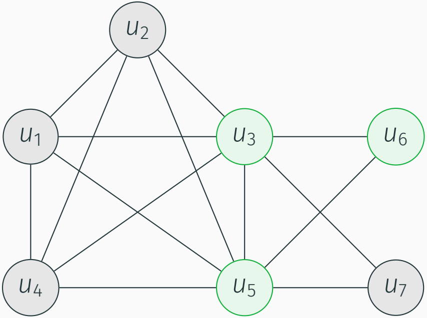

clique of size 3

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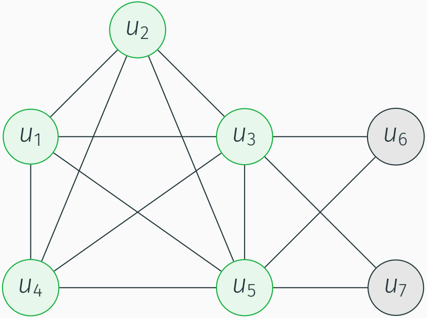

maximum clique of size 5

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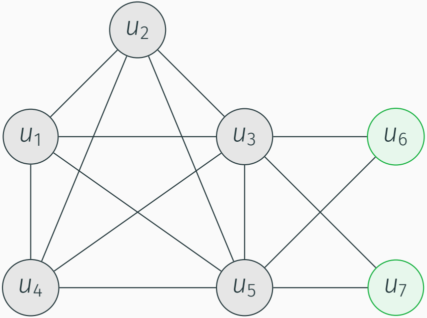

independent set of size 2

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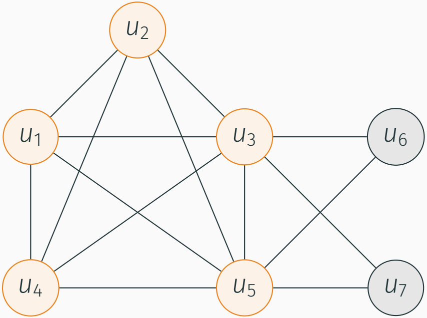
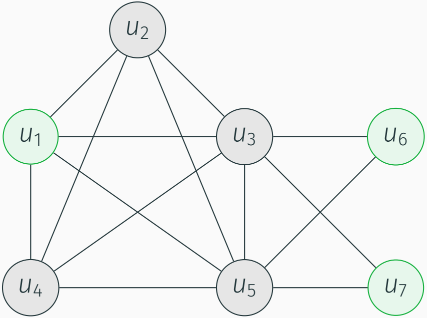


vertex cover of size 5

graph $G = (V, E)$:



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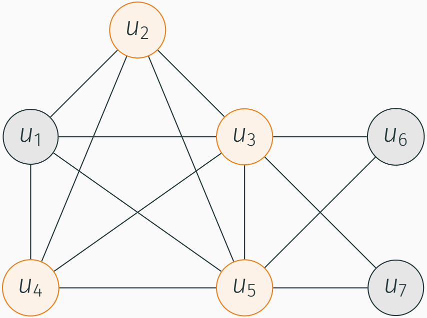

minimum vertex cover of size 4

Hidden pairwise encodings

Standard encoding of maximum clique

construct $\mathcal{F} = \langle \mathcal{H}, \mathcal{S} \rangle$

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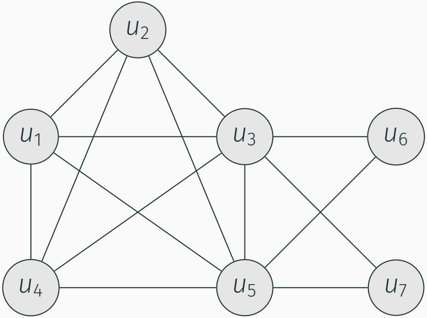

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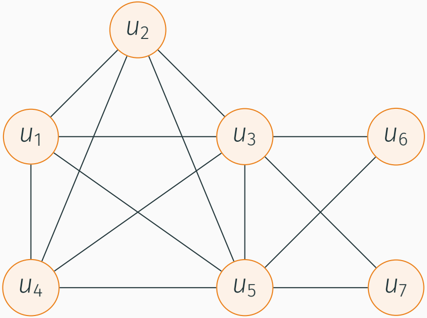

$$\mathcal{S} = \left\{ \begin{matrix} (x_1)\ (x_2)\ (x_3) \\ (x_4)\ (x_5)\ (x_6) \\ (x_7) \end{matrix} \right\}$$

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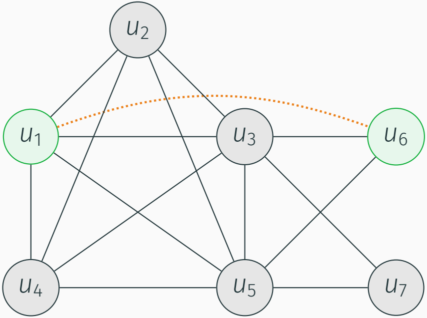

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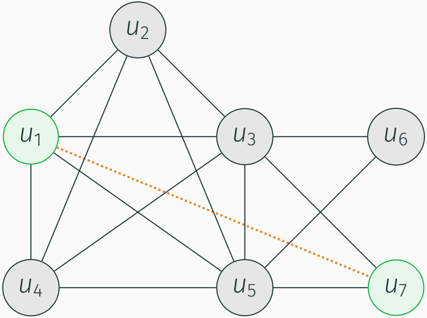

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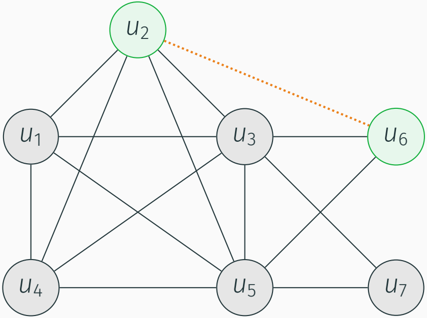

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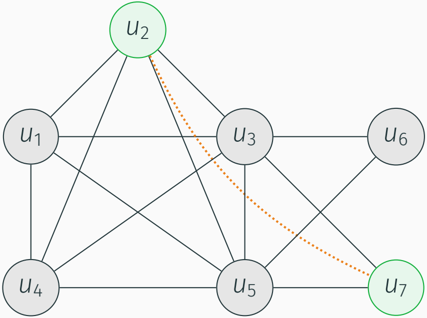

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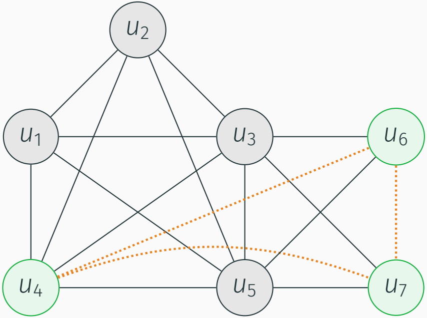

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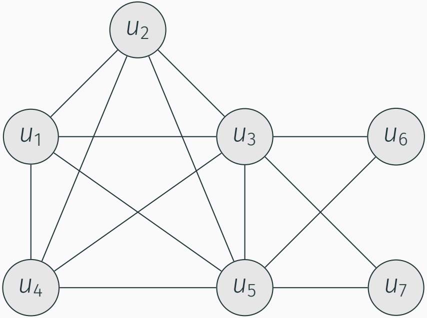

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solve $\mathcal{F}$ with MaxSAT

Hidden pairwise encodings

for ca-dblp-2012[1],

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$|V| = 317\,080$ and $|E| = 1\,049\,867$

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⬇

$$|\mathcal{H}| = |E^C| = 50\,268\,654\,793!$$

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⬇

$|\mathcal{H}| = |E^C| = 50\,268\,654\,793!$

⬇

impossible to solve and **hard** to *represent*

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Edge cover by cliques

$$G = (V, E)$$

$T \subseteq V$ — a clique of G

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➡

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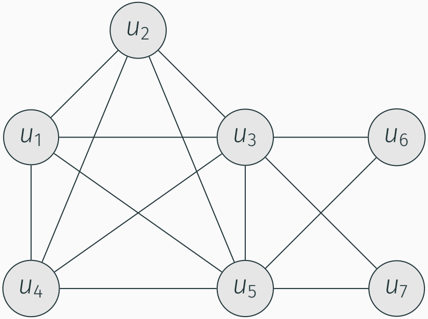

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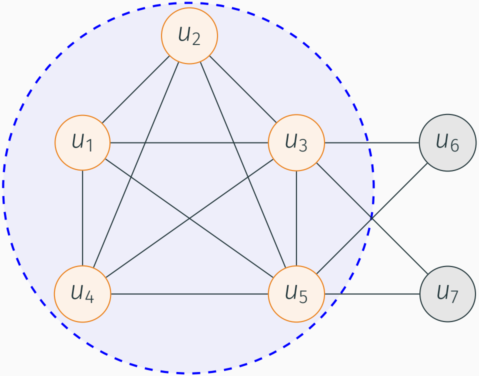

AtMost1(x_1, x_2, x_3, x_4, x_5)

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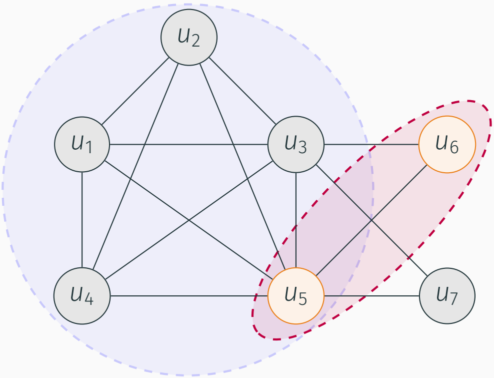

AtMost1(x_5, x_6)

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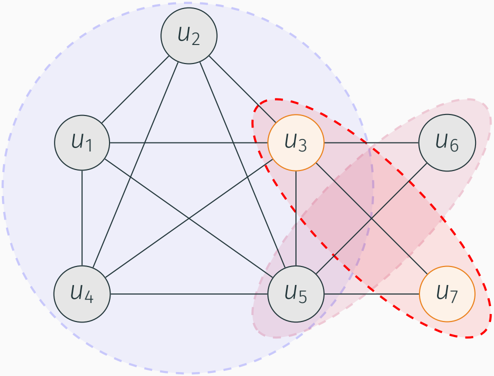

AtMost1(x_3, x_7)
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AtMost1(x_1, x_2, x_3, x_4, x_5)

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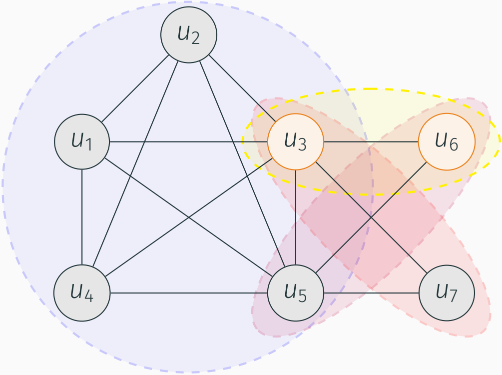

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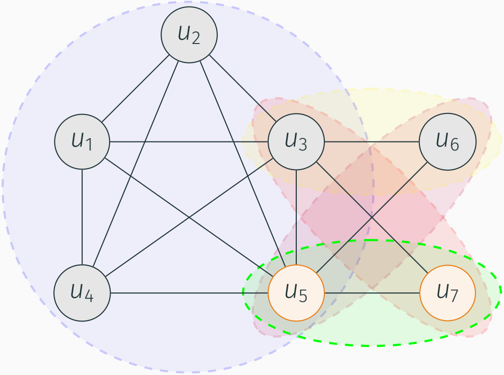

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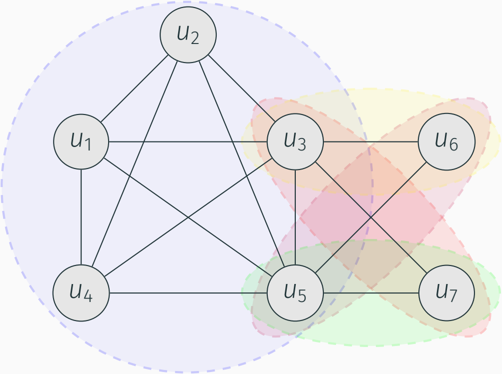

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pairwise encoding!

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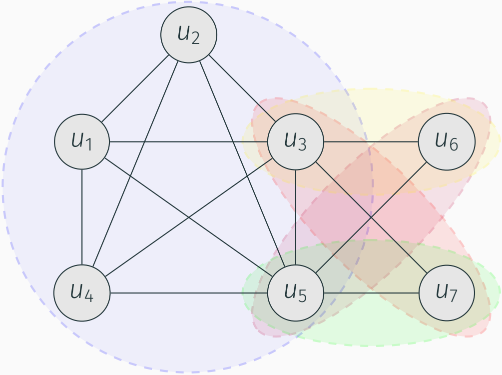

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Solving MaxClique with SAT

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$$\Downarrow$$

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Solving MaxClique with SAT (optimizations)

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⬇

$$\forall u \in V \text{ s.t. } \mathrm{Deg}(u) < K - 1: \quad (\neg x_u)$$

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$$x_u \rightarrow \left(\sum_{v \in \mathrm{Adj}(u) \wedge \mathrm{Deg}(v) \geq K-1} x_v = K - 1 \right)$$

Experimental results

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- Machine configuration:
  - Intel Xeon E5-2630 2.60GHz with 64GByte RAM
  - running Ubuntu Linux
  - 3600s timeout
  - 10GByte memout

Experimental evaluation

Instance	SATClq	Cliquer	FMC	IncMaxCLQ	LMC
comm-n1000	0.19	0.02	0.05	0.11	0.5
comm-n10000	—	0.99	—	12.33	0.93
ca-AstroPh	0	101.17	0.43	—	0.69
ca-citeseer	0	354.46	0.92	—	1.03
ca-coauthors-dblp	0	—	29.65	—	9.42
ca-CondMat	0	71	0.13	—	0.55
ca-dblp-2010	0	353.85	0.87	—	0.92
ca-dblp-2012	0	—	1.39	—	1.07
ca-HepPh	0	44.61	0.57	—	0.6
ca-HepTh	0	27.84	0.06	—	0.49
ca-MathSciNet	0	—	1.27	—	1.07
ia-email-EU	2.47	7.15	0.08	—	0.49
ia-reality-call	0	3.98	0.03	—	0.44
ia-retweet-pol	1.76	2.35	0.16	—	0.49
ia-wiki-Talk	—	60.48	4.21	—	0.73
rt-pol	1.7	2.39	0.19	—	0.49
rt_barackobama	0	0.46	0.45	6.63	0.46
soc-advogato	0.15	0.25	0.11	4.91	0.49
soc-epinions	—	101.67	0.21	—	0.82
soc-gplus	0.01	2.82	0.45	—	0.47
tech-as-caida2007	0.01	5.26	0.09	—	0.48
tech-internet-as	0.02	12.23	0.45	—	0.52
tech-pgp	3.05	0.71	0.07	—	0.45
tech-WHOIS	—	10.13	—	6.31	0.49
web-arabic-2005	0	151.31	2.43	—	1.57
web-baidu-baike-related	0.94	—	—	—	2.54
web-it-2004	0	—	25.32	—	4.87
web-NotreDame	0	—	3.76	—	1.37
web-sk-2005	0	97.44	0.34	—	0.64
p5sparse1	2.88	1031.15	—	12.17	0.48
p5sparse2+10clq20	1.9	24.42	—	—	0.54
p5sparse3+10clq20	3.62	150.15	—	—	0.58
p6sparse1	48.34	—	—	—	0.53
p6sparse2+10clq20	42.65	—	—	—	0.64
p6sparse3+10clq20	50.88	—	—	—	0.7
Solved (out of 35)	31	26	26	6	35

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ca-MathSciNet	0	—	1.27	—	1.07
ia-email-EU	2.47	7.15	0.08	—	0.49
ia-reality-call	0	3.98	0.03	—	0.44
ia-retweet-pol	1.76	2.35	0.16	—	0.49
ia-wiki-Talk	—	60.48	4.21	—	0.73
rt-pol	1.7	2.39	0.19	—	0.49
rt_barackobama	0	0.46	0.45	6.63	0.46
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soc-epinions	—	101.67	0.21	—	0.82
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ia-wiki-Talk	—	60.48	4.21	—	0.73
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