

A Scalable Two Stage Approach to Computing Optimal Decision Sets

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February 4–7, 2021 | **AAAI**

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Problem and state of the art

Problem example

(classification scenario)

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#	Day	Venue	Weather	TV Show	<i>Date?</i>
e_1	Weekday	Dinner	Warm	Bad	No
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must **respect training data** & *generalize well...*

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highly interpretable!

rule-based models

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“transparent” and **easy to interpret**

rule-based models



“transparent” and **easy to interpret**



come in handy in XAI

State of the art — a typical approach

input : training data E

output: *smallest*^a decision set ϕ

```
1  $N \leftarrow LB$                                      #  $N$  equals a lower bound on  $|\phi|$ , which is often set to 1
2 while True:
3      $F \leftarrow \text{Encode}(E, N)$                    # encode problem "is there a decision set  $\phi$  of size  $N$  for data  $E$ ?"
4      $(st, \mu) \leftarrow \text{Oracle}(F)$                # call a reasoning oracle to answer the question
5     if  $st$  is True:
6         break
7      $N \leftarrow N + 1$ 
8  $\phi \leftarrow \text{ExtractRules}(\mu)$                  # extract decision set  $\phi$  from satisfying assignment  $\mu$ 
9 return  $\phi$ 
```

^awrt. the number of rules or literals

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encoding is too large!
(does not scale)

Our approach

divide the process into two stages:

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- MaxSAT-based
- **incremental!**

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each class is computed *independently*

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the idea is to **scale better**

Stage 1 — learning rules

each rule is a **solution to MaxSAT formula**

$$\psi \triangleq H \wedge S$$

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- minimize the number of used literals

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$\mathcal{O}(K + M)$ variables and $\mathcal{O}(K \times M)$ clauses

(K — number of features, M — number of training instances)

Stage 2 — computing rule cover

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$\mathbf{b}_j \in \{0, 1\}$ and $s_j = |\pi_j|$ for each π_j

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$\mathbf{A} = (\mathbf{a}_{ij}), \mathbf{a}_{ij} = 1$ iff π_j covers e_i

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	π_1	π_2	π_3	π_4
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s_j	1	1	1	1

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	π_1	π_2	π_3	π_4
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minimize $\sum_j s_j \cdot b_j$

subject to $\sum_j a_{ij} \cdot b_j \geq 1, \forall i$

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for each rule, add **one clause enforcing**
all following rules to **cover ≥ 1 other instance**

Experimental results

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- **machine configuration:**
 - Intel Xeon Silver-4110 2.10GHz with 64GByte RAM

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 - **3–384 features** (one-hot encoded)
 - **14–67557 training** instances

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- **competition tested:**
 - **m_{ds}₂** – minimization of number of rules

¹<https://github.com/alexeyignatiev/minds>

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 - **same code base** and SAT solver – Glucose 3

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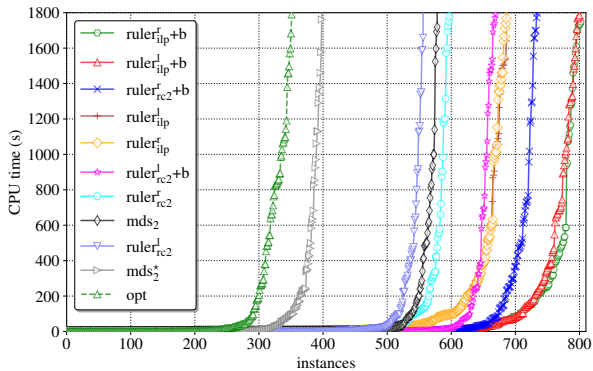
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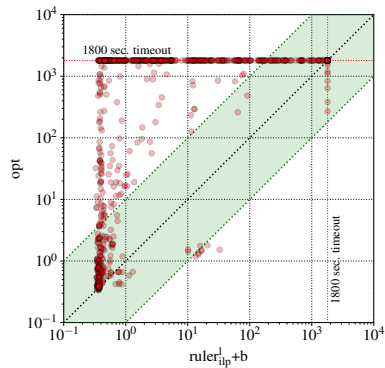
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 - **ruler_{*}^o+b** – symmetry breaking *enabled*

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Results – performance comparison

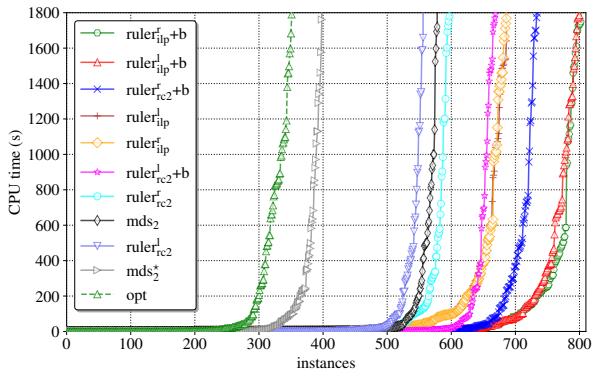


(a) raw performance

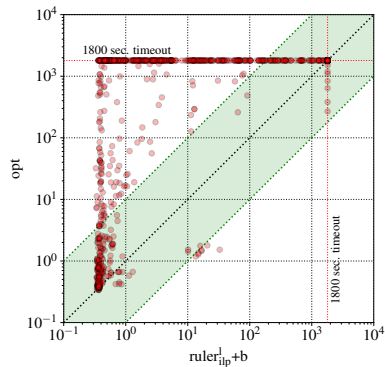


(b) ruler^l_{ilp}+b vs. *opt* detailed

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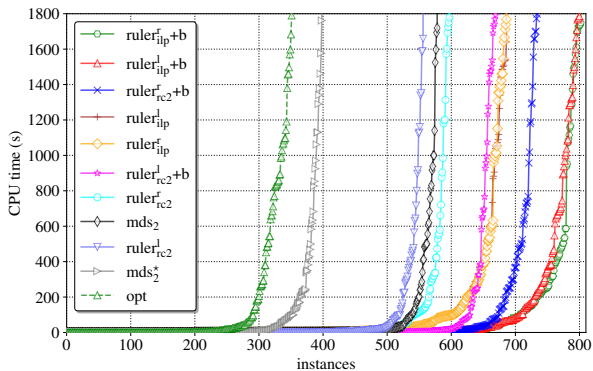
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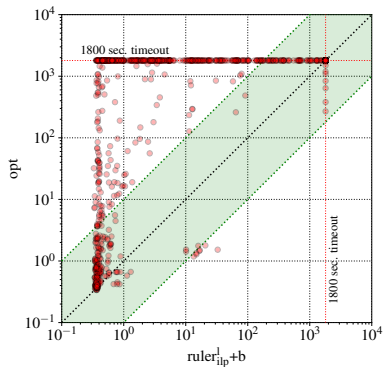
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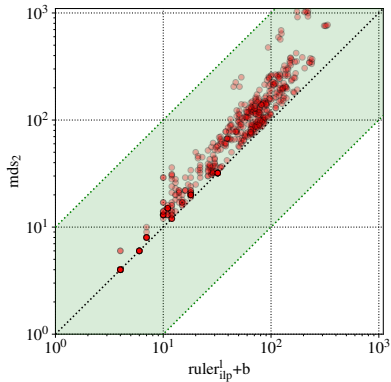


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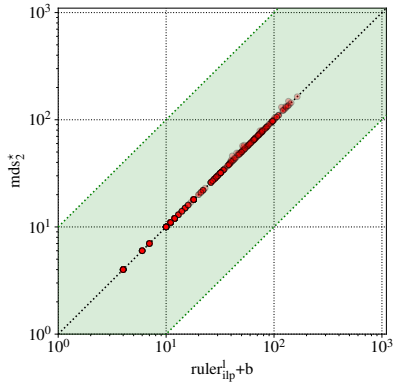
ruler^l_{ilp}+b vs. *opt* — **up to 4 orders of magnitude** performance improvement

breaking symmetric rules — avg. # of rules goes down **from 19604.4 to 563.7**

Results – model size comparison

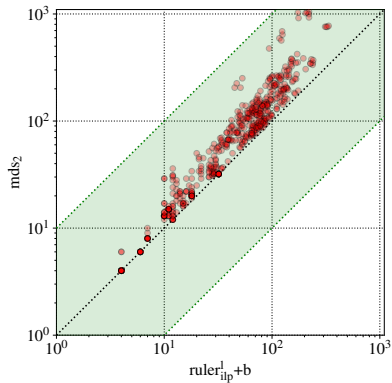


(a) literals or rules: $\text{ruler}_{ilp}^l + b$ vs. mds_2

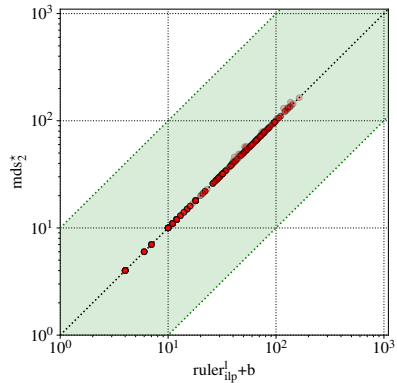


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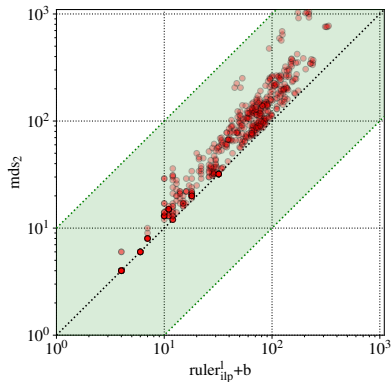
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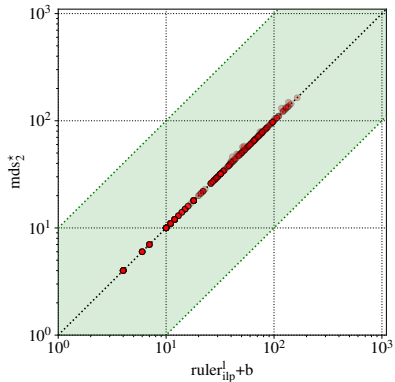
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mds_2^* vs. mds_2 — *lexicographic optimization* pays off
(but slower!)

Summary and future work

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 2. solving **set cover** problem
 - **effective symmetry breaking**
 - **smallest size** decision sets wrt.
 - number of rules
 - total number of literals

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 - total number of literals
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- other rule-based models:
 - decision lists
 - decision trees

Questions?